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An output-only stochastic parametric approach for the identification of linear and nonlinear structures under random base excitations: Advances and comparisons

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A B S T R A C T

In this paper a time domain output-only Dynamic Identification approach for Civil Structures (DICS) first formulated some years ago is reviewed and presented in a more generalized form. The approach in question, suitable for multi- and single-degrees-of-freedom systems, is based on the statistical moments and on the correlation functions of the response to base random excitations. The solving equations are obtained by applying the Itô differential stochastic calculus to some functions of the response. In the previous version ([21] Cavaleri, 2006; [22] Benfratello et al., 2009), the DICS method was based on the use of two classes of models (Restricted Potential Models and Linear Mass Proportional Damping Models) while its generalization for use with different models from the ones mentioned above is discussed. In the paper the new class of models to which the DICS method is applicable are described. Further, the advantages and disadvantages of the approach in question are examined, also by a comparison with some techniques available in the literature.

1. Introduction

Many identification techniques are based on knowledge of input (in a deterministic or probabilistic sense) both in the field of linear system identification (e.g. [1–4]) and in the field of nonlinear system identification (e.g. [5–7]). Unfortunately, input is not always available, as in the case of environmental excitations. In some cases it is even unmeasurable. As an example, for a ship rolling in waves the actual wave moment experienced by the ship when it is moving is not measurable. However, not being constrained to measure the input, even when it is measurable, is an advantage in any case. Awareness of this has increased interest in the so-called output-only identification techniques. In this field, referring to state invariant systems some interesting approaches have been proposed in the past (e.g. [8–16]). Nevertheless, in many cases, the approaches proposed refer to single-degree-of-freedom systems, more frequently linear or weakly nonlinear, or show computational difficulties in the case of multi-degrees-of-freedom systems; in some cases the damping estimation depends on a priori knowledge of the input, evidencing a non-negligible limit.

Another question regards the linearity and the nonlinearity of the systems to be identified. Currently the literature shows that researchers maintain a certain interest in linear systems whose identification is simpler to obtain. For example Ceravolo and Abbiati [15] recently discussed some identification techniques (in detail methods based on time series autoregressive (AR) models are compared with the eigensystem realization algorithms (ERA) applied to random decrement signatures and stochastic subspace identification methods (SSI)). Besides, despite the fact that the above techniques are suitable for linear systems, in [15] it is shown that they require long time responses and in some cases show high errors, mainly in the evaluation of the damping characteristics.

Within the context of output-only identification techniques, Spiridonakos et al. [16] have also been interested in the identification of linear systems: in this case time-varying systems are considered and the possibility of their identification is discussed by means of three different approaches based on the parametric mathematical models of time-dependent autoregressive moving averages (TARMA). These approaches are compared in terms of frequencies referring to a steel beam clamped close to its ends on vertical stands in a laboratory, considering a mass sliding on the above steel beam.

In [17] too the interest is limited to linear systems, and further, in this case, the problem of stiffness characterization is faced neglecting the problem of the identification of the dissipation characteristics.

The identification of nonlinear systems presents major difficulties that cannot always be overcome. For example in [8] an approach is

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proposed for signal decomposition and nonlinearity identification in the time domain based on the empirical mode decomposition method (EMD) integrated by the use of a pair of sliding conjugate functions, but its effectiveness has been proved in the case of a weakly nonlinear Duffing oscillator under a harmonic excitation having a frequency equal to the linear frequency of the oscillator in question and in the case of free vibrations. In the work in question the difficulties encountered in the identification of the parameter characterizing both linear dissipation force and restoring forces are pointed out, showing the failure of that technique in the case of strong nonlinearities.

The nonlinearities to be identified are limited to the restoring forces in [18] where an output-based approach is proposed which uses empirical mode decomposition (EMD).

Further techniques in the nonlinear field are discussed in the extended review proposed by Kerschen et al. [19], who underline the difficulty of developing a method with broad applicability but also the need to formulate techniques suitable not only in the case of weak nonlinearities, to which most of the available techniques are addressed.

In this framework, improvement of the available techniques or the formulation of new ones is a goal to be reached.

Here, a time domain approach, proposed some years ago, is reviewed and proposed again in an advanced generalized formulation (DICS). In detail, this approach was applied in the past to two specific classes of models and specific solving equations for the classes of models considered were given [20–22]. The validity of the DICS method could not be proved for classes different from those mentioned above. Now the DICS method is presented in a general formulation not depending on the classes of models considered in the past, showing its capacity to overcome the limit presented in the original formulation and the possibility of its being applied to a larger number of models.

The technique in question has proved to be suitable for the identification of SDOF and MDOF systems under unavailable white noise input. Further, this technique effectively overcomes some limits encountered in the techniques available in the literature. For example, it gives the possibility of capturing either linearity and nonlinearity without loss of reliability and of capturing either weak and strong nonlinearities of MDOF systems, as will be discussed in the next sections. The formulation here proposed does not claim to solve all problems arising in system identification, especially in nonlinear identification, but aims to make a contribution to overcoming some longstanding difficulties.

The solving equations are obtained by using *Itô* calculus [23] through three steps. The stiffness parameters are obtained in the first one, while in the second and in the third the input and the dissipation parameters are obtained.

As mentioned before the DICS method was initially formulated to be used with only two classes of models, that is Restricted Potential (non-linear) Models, as defined in [24–26], and classical linear models with Mass Proportional Damping [27]. The advantage of using these models is that the probability density functions (pdf) of the state variables can be obtained in an exact analytical form. The availability of the pdf has made it possible to prove some properties that have allowed to obtain the identification algorithm in a special form, as will be better explained in the next sections. Here, that algorithm is presented in a form suitable for a more extended class of linear and nonlinear systems, namely in a generalized form that makes it more attractive for practical applications.

The approach discussed here refers to time invariant systems but this does not preclude an extension to the case of time variant mechanical characteristics.

In the paper comparisons between the results obtainable with the proposed generalized approach and the results obtainable with other techniques available in the literature are discussed.

2. The algorithm proposed for parameter identification

In the proposed procedure the identification problem consists of searching for the best model of a system, invariant at least during the experiment, whose response is observable (observable means that at least the acceleration can be measured and consequently velocity and displacement can be analytically obtained). The model is completely defined and ready for structural analysis when the values of the parameters of the restoring and of the damping forces are estimated.

As mentioned before, the strategy for deriving the DICS algorithm, in the version proposed in [21,22], depended on the use of specific classes of models. In detail Restricted Potential Models (RPM) and Mass Proportional Damping Models (MPDM) were used. Here a generalized form of the DICS algorithm is proposed, also to be used with classes of models different from RPM and MPDM.

In order to derive the solving algorithm reference is made to any dynamical system whose govern equation, normalized with respect to masses, is

$$\ddot{\boldsymbol{X}}+\tilde{\boldsymbol{D}}(\boldsymbol{X},\dot{\boldsymbol{X}})+r(\boldsymbol{X})=\tilde{\boldsymbol{W}} \tag{1}$$

In Eq. (1) $\boldsymbol{X}$ is the n-dimensional displacement vector, the upper dot means time derivative, $\boldsymbol{r}(\boldsymbol{X})$ is any vector of nonlinear functions representing the restoring forces, and $\tilde{\boldsymbol{D}}(\boldsymbol{X},\dot{\boldsymbol{X}})$ is a vector of dissipation forces given as

$$\tilde{\boldsymbol{D}}(\boldsymbol{X},\dot{\boldsymbol{X}})=\tilde{\boldsymbol{C}}\frac{\partial}{\partial\dot{\boldsymbol{X}}}\Delta(H). \tag{2}$$

In Eq. (2), $\Delta(\cdot)$is a function of the total energy H of the system, H is expressed as

$$H=\frac{1}{2}\dot{\boldsymbol{X}}^2+U(\boldsymbol{X});\ \frac{\partial U(\boldsymbol{X})}{\partial\boldsymbol{X}_i}=r_i(\boldsymbol{X}), \tag{3}$$

$\tilde{\boldsymbol{C}}$ is a matrix of parameters, and $(\partial/\partial\dot{\boldsymbol{X}})^T=((\partial/\partial\dot{\boldsymbol{X}}_1),.....,(\partial/\partial\dot{\boldsymbol{X}}_n))$. In Eq. (3) $r_i(\boldsymbol{X})$ is the i-th entry of the vector of the restoring forces. Finally, $\tilde{\boldsymbol{W}}(t)$ (the external input) is a vector of zero mean white noise processes characterized by the correlation matrix $\tilde{\boldsymbol{R}}$ whose ij-th term $\tilde{R}_{ij}$ is

$$\tilde{R}_{ij}=2\pi\tilde{K}_{ij}\delta(\tau)=E[\tilde{W}_i(t)\tilde{W}_j(t+\tau)] \tag{4}$$

In Eq. (4) $E[\cdot]$ is the average operator, t means time, τ is a time delay, and $\delta(\tau)$is the Dirac's delta. Moreover, $\tilde{K}_{ij}$ is the $\boldsymbol{ij}$-th term of the matrix $\boldsymbol{K}$, which is the Power Spectral Density matrix of $\tilde{\boldsymbol{W}}$.

Eq. (1) refers to linear or nonlinear dynamical systems both in terms of restoring and dissipation forces. It gives a classical linear system if $\boldsymbol{r}(X)=\boldsymbol{RX}$ and $\Delta(H)=H$, and further it includes the RPM and MPDM defined in [21,22].

Eq. (1) can be rewritten in the *Itô* form simply by setting $\boldsymbol{Z}_1=\boldsymbol{X}$, $\boldsymbol{Z}_2=\dot{\boldsymbol{X}}$, that is

$$d\boldsymbol{Z}_1=\boldsymbol{Z}_2dt \tag{5a}$$

$$d\boldsymbol{Z}_2=-\tilde{\boldsymbol{D}}(\boldsymbol{Z}_1,\boldsymbol{Z}_2)dt-r(\boldsymbol{Z}_1)dt+d\tilde{B} \tag{5b}$$

where $\tilde{\boldsymbol{B}}$ is the vector of the Wiener processes whose time formal derivative is the white noise vector $\tilde{\boldsymbol{W}}(t)$, that is $(d\tilde{\boldsymbol{B}}_i/dt)=\tilde{\boldsymbol{W}}_i(t)$.

In the case of structures under base excitation, the input in Eq. (1) can be rewritten in the form:

$$\tilde{\boldsymbol{W}}=\boldsymbol{L}W_0 \tag{6}$$

$\boldsymbol{L}$ being the n-dimensional influence vector that in the case of plane behavior assumes the form

$$\boldsymbol{L}^T=[1,1,........,1] \tag{7}$$

and $W_0=dB_0/dt$ being the white noise base input whose power spectral density is K_0. In the case of base excitation each term of