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Maximum roll angle estimation of a ship in confused sea waves via a quasi-deterministic approach

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ABSTRACT

This paper considers the maximum roll motion of a ship in confused sea waves. The ship motion is described by a nonlinear differential equation including quadratic damping and cubic restoring force. The excitation of the ship is represented by a stationary mean-zero Gaussian process of a given power spectral density function. It is shown that a reliable estimate of the maximum roll motion is found considering the ship response to an approximate deterministic representation of an appropriately large and adequately rich (frequency-wise) load. Specifically, the time variation of the load is approximated by a normalized autocovariance function; the maximum amplitude of the load is taken as a certain multiple of the standard deviation of the stochastic load process. This approximation relates to the method of quasi-deterministic representation of extreme realizations of a stationary Gaussian process; the method is interpreted as a tool for generating deterministic time histories of the load which are compatible with a certain power spectral density function. The efficacy of this perspective is shown by comparison with the results from pertinent Monte Carlo simulations.

Next, the paper addresses the ship stability problem in the space of initial conditions. In this context, it shows that the proposed approximation can be adequately utilized for a ship safety assessment.

1. Introduction

A safety and reliability based design procedure of ships relies on several approximate techniques for the determination of the ship response to random sea waves. These techniques involve the modeling of a six-degree-of-freedom system subject to random loads. In ship dynamics terminology, these degrees of freedom are commonly referred to as heave, sway, surge, yaw, pitch and roll (Fig. 1). One of the fundamental requirements in ship design is ensuing stability against capsizing. In this regard, the most critical degree of freedom is roll [1].

The roll angle estimation can be pursued by simplified models. Specifically, the roll is modeled as a single degree of freedom (SDOF) oscillator, where nonlinearities are included. The nonlinearities are due to the restoring moment, utilized for keeping the ship in a stable position, and to the viscous damping, that relates to the fluid behavior on the ship boundary. More specifically, the former relates to the shape of the righting arm diagram. This diagram shows that the restoring moment is a nonlinear function of the roll angle. Thus, it is often approximated by a polynomial of odd powers, which yields nonlinear stiffness terms [2]. Further, the latter relates to the viscosity and to the generation of vortices. The damping term is not derived by analytical arguments. Instead, it is added, phenomenologically, to the equation of motion. Two models are commonly utilized: a drag-type model which includes quadratic damping; and a cubic model [3]. Accounting for realistic situations induces the use of a stochastic model of the excitation. In this regard, the most commonly used model, the sea state theory [4], represents the free surface displacement by a stationary Gaussian process [5], so that, in linear water waves, the hydrodynamic excitations are stationary Gaussian processes, as well.

The Monte Carlo simulation technique is, currently, the most versatile method for determining the system response statistics [6,7], as nonlinearities are readily accounted for. In this regard, the common procedure for calculating response statistics is to synthesize an adequately long time history of the system's input which is compatible with a specified power spectral density. Then, the response is determined by numerical integration of the equation of motion. In the context of ship dynamics, detailed analysis of extreme response in the time domain is required as well [8], because severe operational conditions may become critical for the ship performance. Obviously, Monte Carlo methods exhibit a remarkable drawback: computational cost. Indeed, they are time consuming even with the advent of the available computational power. This disadvantage is due to the need of quantifying events

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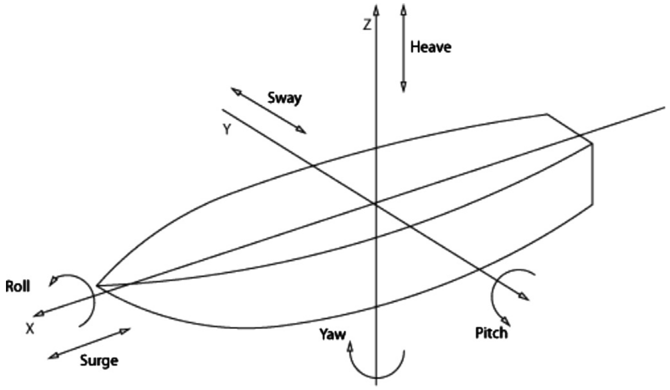

Fig. 1. Six degrees of freedom of a ship model in a schematic diagram.

with a quite low probability of occurrence. Solutions to this problem have been pursued by various authors. For example, Naess and Gaidai [9] assumed a simplified framework in which the statistical distribution of extreme values is determined by the mean upcrossing rate. Thus, they adopted a Poisson assumption: upcrossing events are independent from each other. This assumption is quite reasonable in many practical situations. However, it is not reliable for narrow band processes. In this regard, different procedures have been developed for considering bandwidth effects [10]. Also, importance sampling techniques have been proposed for reducing time computation with respect to Monte Carlo simulations [11–13].

Various research efforts have focused on the calculation of the probability density function (pdf) of the response. In this case, difficulties arise due to the actual inability to determine the exact probability density function solution of a nonlinear problem. Thus, approximate techniques have been pursued. The most used approximation relies on the equivalent linearization concept [14,15]. By this technique, the nonlinear roll motion equation is replaced by a linear equation of motion which is equivalent to the nonlinear one according to a certain criterion. A different approach involves the solution of the associated Fokker–Plank–Kolmogorov (FPK) Eq. [16]. In this context, finite-element methods have been proposed by Spencer and Bergman [17] and an approximate solution was developed by Er and Iu [18]. Other methods were developed, as well. For instance, the Gaussian closure method [19] and the stochastic average method [20].

This paper presents an approximate approach for estimating the maximum roll angle of a ship in a confused sea. The ship is modeled as a nonlinear oscillator subject to a stationary mean-zero Gaussian load of a given power spectral density. The approach relies on a quasi-determinism based representation of the wave load [21]. Such a representation approximates the temporal variation of the dynamic load by the variation of the normalized autocovariance function of the load process, in the vicinity of a large maximum. The concept has been used for several years in the context of probabilistic wave mechanics. For instance, it has been utilized for approximating the space-time variation of the free surface displacement close to a quite large wave height [21–23]. Herein, this formalism is used as a tool for synthesizing realizations of a "large" ship excitation, which are compatible with a specified spectrum. The usefulness and reliability of the method is assessed by comparison with the results of Monte Carlo simulations.

Next, the paper addresses the ship stability problem in space of initial conditions [24]. Specifically, it shows that integrity curves can be constructed to identify the critical wave excitation amplitude associated to a certain degree of "erosion" of the safe basin. In this context, an interesting feature of the proposed formulation relates to the possibility of generating a quasi-impulsive time variation of the excitation which is spectrum compatible.

2. Theoretical background

2.1. Mathematical description of the roll motion

The ship roll motion equation is formulated following an approximation utilized in several studies [25,26]. Specifically, the motion is assumed to be governed, approximately, by the nonlinear equation:

$$\ddot{\varphi}+F(\dot{\varphi})+G(\varphi)=M(t) \tag{1}$$

in which $\ddot{\varphi}$ is the roll angle, $F(\dot{\varphi})$ denotes the nonlinear damping, $G(\varphi)$ denotes the nonlinear stiffness, and $M(t)$ is the wave exciting moment. The damping term relates to viscous and pressure drags generated by the relative velocity between flow field and structure. In the following, a linear-plus-quadratic model is utilized. The stiffness term $G(\varphi)$ relates to the restoring moment of the ship. Specifically, it captures the righting arm dependence on the roll angle (Fig. 2). The mathematical treatment is simplified by approximating the diagram by an odd-order polynomial. In the following, a linear-plus-cubic approximation is utilized. This approximation is reliable only for moderate values of the roll angle ($<35^\circ$ [27]), but it is widely utilized since ships are not ordinarily expected to experience larger angles of motion. Thus, the equation of roll motion (1) is recast in the form

$$\ddot{\varphi}+a_1\dot{\varphi}+a_2\dot{\varphi}|\dot{\varphi}|+a_3\varphi+a_4\varphi^3=M(t), \tag{2}$$

where the a_i-coefficients ($i=1$, 2, 3, and 4) depend on the ship characteristics and can be estimated by system identification procedures [25]. The wave exciting moment $M(t)$ is modeled as a stationary Gaussian process of a given power spectral density. This assumption is consistent with the standard representation of the random free surface displacement in linear water waves [4]. Indeed, the power spectral density of $M(t)$ is commonly determined from the free surface displacement spectrum via the calculation of a transfer function pertaining to the specific ship model under consideration. In this regard, the spectrum of $M(t)$ can be approximated quite satisfactorily by a filtered white noise [28], as long as the excitation is a stationary process. The approximating dimensionless spectrum is given by the following equation:

$$\hat{S}_M(w)=\frac{Gw^4}{[(w^2-k_1)^2+(c_1w)^2][(w^2-k_2)^2+(c_2w)^2]} \tag{3}$$

where G, k_1, k_2, c_1, c_2 are parameters determined by fitting the target spectrum by a least-square algorithm, and w is an appropriately scaled (dimensionless) frequency ($\equiv \omega T_n/2\pi$; being ω radian frequency and T_n undamped natural frequency). Eq. (3) can

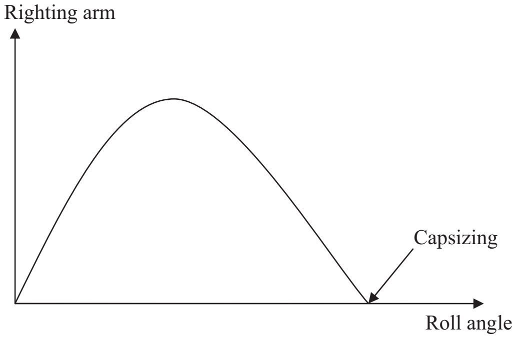

Fig. 2. Righting arm versus roll angle.