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Explicit sensitivities of the response of discretized structures under stationary random processes

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ABSTRACT

This study presents a semi-analytical approach for the sensitivity analysis of the response of linear discretized structures subjected to stationary multi-correlated Gaussian random excitation. The proposed procedure relies on the use of the so-called *rational series expansion* (*RSE*), recently derived by the authors for evaluating in approximate explicit form the inverse of a matrix with small rank-*r* modifications. The *RSE* allows to determine the mean-value and power spectral density function of the response as approximate explicit functions of the design parameters. Direct differentiation of these functions with respect to the design parameters provides approximate analytical expressions of the sensitivities of the probabilistic characteristics of the stationary stochastic response in the frequency domain. Numerical results concerning different structural systems under random excitation are presented to demonstrate the accuracy and efficiency of the proposed procedure.

1. Introduction

The sensitivity analysis is a very useful tool for predicting the variation of the structural response due to changes in design parameters. It deals with the calculation of changes in the response resulting from changes in the parameters describing the structure and it basically relies on the evaluation of partial derivatives of a performance measure with respect to system parameters (see e.g. [1–3]).

The partial derivatives of the response with respect to the design variables, also referred to as sensitivity coefficients, are used in the solution of various problems: in design optimization, these coefficients are employed to select a search direction in order to optimize the performance under service loads as well as to reduce global construction costs [4]; in the context of reanalysis, the sensitivity coefficients are exploited to generate approximations of the response of a modified system, including approximate reanalysis models and explicit approximations of the constraint functions in terms of structural parameters [5]. Moreover, the sensitivities are often required for assessing the effects on the system response of uncertain structural parameters (e.g. material or geometrical properties) ever present in structural engineering problems [6]. Indeed, it is now widely recognized that a deterministic treatment of input parameters may lead to unreliable predictions of structural behavior. In this context, it is valuable for design purposes to know the impact of each uncertain input variable on the behavior of a structure.

The main dynamic excitations arising from natural phenomena, such as earthquake ground motion, gusty winds or sea waves, are commonly modeled as Gaussian stochastic processes for structural analysis purposes. In the framework of Stochastic Mechanics, several approaches have been proposed to cope with the challenging problem of characterizing the random response of a structural system under stochastic excitation. In particular, it is well-known that the random response is fully defined from a probabilistic point of view by the knowledge of its probability density function (PDF). Moreover, if the system has a linear behavior and it is forced by a Gaussian random process, the response is Gaussian too. In this case, the probabilistic characterization of the stochastic response can be performed either in the so-called time domain by evaluating the mean-value vector and the correlation function matrix, or in the so-called frequency domain through the knowledge of the mean-value vector and the power spectral density (PSD) function matrix (see e.g. [7,8]).

The expectations and covariances of the sensitivity coefficients of the response of randomly excited structures have been earlier evaluated by Socha [9] and Szopa [10]. More recently, Benfratello et al. [11] proposed a time domain approach for evaluating the sensitivity of the statistical moments of the response of structural

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systems under stationary Gaussian and non-Gaussian white input processes. Bhattacharyya and Chakraborty [12] extended the Neumann expansion method within the framework of Monte Carlo simulation for sensitivity analysis of dynamical systems subjected to ground acceleration modeled as a stationary random process. Chaudhuri and Chakraborty [13] dealt with the evaluation of response sensitivity in the frequency domain of structures subjected to non-stationary seismic excitation. Cacciola et al. [14] proposed a semi-analytical approach for determining the sensitivity coefficients of the stochastic response of both classically and non-classically damped structural systems subjected to stationary and non-stationary stochastic Gaussian excitation. Marano et al. [15] performed the stochastic sensitivity analysis of the response with respect to uncertain soil parameters in the presence of a non-stationary seismic excitation.

As stated by Jensen [16], the approaches for calculating the partial derivatives of a performance measure with respect to the system parameters can be classified into analytical, numerical and semi-analytical. Analytical methods evaluate analytically the sensitivity coefficients but are sometimes difficult to implement in a given numerical method. Conversely, numerical approaches involve the computation of the derivatives by the finite difference method so that the implementation is very easy but the accuracy and the computational costs are not competitive. Finally, the semi-analytical approaches are based on the evaluation of the derivatives of response quantities by an expansion of the response (e.g. first-order perturbation, first-order Taylor expansion, etc.). It follows that they combine the easiness of implementation of numerical approaches and efficiency of analytical methods.

This study presents a novel semi-analytical approach for the sensitivity analysis of the response of linear discretized structures subjected to stationary multi-correlated Gaussian random excitation. The main feature of the procedure is that it provides very accurate approximate analytical expressions of the sensitivities of the mean-value and PSD function of the stationary response. This remarkable result is achieved by using the so-called *rational series expansion* (*RSE*), recently proposed by the authors [17–19] as an alternative explicit form of the Neumann series expansion of the inverse of an invertible matrix with small rank-r modifications. Indeed, the *RSE* allows to obtain approximate explicit expressions of both the mean-value and PSD function of the response in terms of the design parameters. Based on the knowledge of the analytical relationships between the response quantities and the system parameters, the sensitivities can be evaluated analytically via direct differentiation. The proposed procedure is easy to implement in a given numerical method such as the finite element method. It can be straightforwardly extended to investigate the effects of uncertain parameters on the stationary response of randomly excited structures. Furthermore, the procedure can also be extended to the case of non-stationary excitation once the stochastic analysis of the response is performed in the so-called *mixed time-frequency domain*.

The objective of numerical results presented in the paper is twofold. On one hand, the accuracy of the explicit sensitivities of the probabilistic characteristics of the response is demonstrated. On the other hand, when a large number of design variables are involved, the usefulness of sensitivity information to improve the computational efficiency of the *RSE* is shown.

The paper is organized as follows: the basic equations governing the probabilistic characterization and sensitivity analysis of the response of linear discretized structures under stationary multi-correlated Gaussian stochastic excitation are recalled in Section 2; Section 3 outlines the evaluation of approximate explicit expressions of both the inverse of the stiffness matrix and the modal *frequency response function* (*FRF*) matrix by means of the *RSE*; in Section 4, explicit expressions of the sensitivities of the mean-value and PSD function of the random response are derived; finally, Section 5 presents numerical results concerning the sensitivity analysis of a truss structure and a framed structure with tuned mass damper under wind excitation modeled as a stationary multi-correlated Gaussian random process; concluding remarks are stated in the last section.

2. Problem formulation

2.1. Equations of motion

The design sensitivity analysis deals with the evaluation of the change in the system response due to design parameter variations in the neighborhood of prefixed values, called nominal parameter values. Let r be the number of significant design parameters whose influence on the response has to be assessed. The jth parameter, P_j, is expressed as $P_j = P_{0,j}(1+\alpha_j)$ where α_j denotes the dimensionless fluctuation around the nominal value $P_{0,j}$. Hence, the nominal configuration of the structure corresponds to the condition $\alpha_j = \alpha_{0,j} = 0$ which yields $P_j \equiv P_{0,j}$, $j = 1,2,\ldots,r$. In structural engineering applications, the dimensionless fluctuations α_j can be reasonably assumed to satisfy the condition $|\alpha_j| < 1$, with the symbol $|\bullet|$ denoting absolute value. Let, $\boldsymbol{\alpha} = [\alpha_1, \ \alpha_2, \ \ldots, \ \alpha_r]^T$ be the r-order vector collecting the r dimensionless fluctuations α_j of the design parameters P_j; the apex T means transpose. In the most general case, the mass, damping and stiffness matrices of the structure and the response vectors should be considered as functions of the design parameters. Based on this observation, the equations of motion of a quiescent $n-$DOF linear structural system subjected to a stationary multi-correlated Gaussian stochastic process $\mathbf{f}(t)$ can be cast in the form:

$$\mathbf{M}(\boldsymbol{\alpha})\,\ddot{\mathbf{u}}(\boldsymbol{\alpha},t)+\mathbf{C}(\boldsymbol{\alpha})\dot{\mathbf{u}}(\boldsymbol{\alpha},t)+\mathbf{K}(\boldsymbol{\alpha})\mathbf{u}(\boldsymbol{\alpha},t)=\mathbf{f}(t) \tag{1}$$

where $\mathbf{M}(\boldsymbol{\alpha})$, $\mathbf{C}(\boldsymbol{\alpha})$ and $\mathbf{K}(\boldsymbol{\alpha})$ are the $n \times n$ mass, damping and stiffness matrices of the structure, respectively; $\mathbf{u}(\boldsymbol{\alpha},t)$ is the stationary Gaussian vector process of displacements and a dot over a variable denotes differentiation with respect to time t. Notice that in the case of seismic excitation, if the mass matrix depends on the design parameters P_j, the forcing vector shall depend on the same parameters (see Appendix A).

Without loss of generality, the $n \times n$ order structural matrices introduced above can be expressed as linear functions of the dimensionless design parameters α_j, i.e.

$$\mathbf{K}(\boldsymbol{\alpha})=\mathbf{K}_0+\sum_{j=1}^{r_K}\alpha_j\mathbf{K}_j,\quad \mathbf{M}(\boldsymbol{\alpha})=\mathbf{M}_0+\sum_{j=1+r_K}^{r_K+r_M}\alpha_j\mathbf{M}_j,$$
$$\mathbf{C}(\boldsymbol{\alpha})=\mathbf{C}_0+\sum_{j=1+r_K+r_M}^{r}\alpha_j\mathbf{C}_j, \tag{2a–c}$$

where

$$\mathbf{M}_0=\mathbf{M}(\boldsymbol{\alpha})\big|_{\boldsymbol{\alpha}=\mathbf{0}};\quad \mathbf{M}_j=\frac{\partial\mathbf{M}(\boldsymbol{\alpha})}{\partial\alpha_j}\bigg|_{\boldsymbol{\alpha}=\mathbf{0}},$$
$$\mathbf{C}_0=\mathbf{C}(\boldsymbol{\alpha})\big|_{\boldsymbol{\alpha}=\mathbf{0}};\quad \mathbf{C}_j=\frac{\partial\mathbf{C}(\boldsymbol{\alpha})}{\partial\alpha_j}\bigg|_{\boldsymbol{\alpha}=\mathbf{0}},$$
$$\mathbf{K}_0=\mathbf{K}(\boldsymbol{\alpha})\big|_{\boldsymbol{\alpha}=\mathbf{0}};\quad \mathbf{K}_j=\frac{\partial\mathbf{K}(\boldsymbol{\alpha})}{\partial\alpha_j}\bigg|_{\boldsymbol{\alpha}=\mathbf{0}}. \tag{3a–f}$$

In the previous equations, $\mathbf{M}_0$, $\mathbf{K}_0$ and $\mathbf{C}_0$ denote the mass, stiffness and damping matrices, respectively, pertaining to the nominal configuration (where the design parameters are set equal to their nominal values, i.e. $\boldsymbol{\alpha}=\boldsymbol{\alpha}_0=\mathbf{0}$) which are positive definite symmetric matrices of order $n \times n$; furthermore, $\mathbf{M}_j$, $\mathbf{K}_j$ and $\mathbf{C}_j$ are semi-positive definite symmetric matrices of order $n \times n$. Without loss of generality, in Eq. (2) the fluctuations of the design parameters