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A convenient approach for estimating time-dependent structural reliability in the load space

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ABSTRACT

A procedure, formulated in the space of the load processes, is described for estimating the reliability of structures subject to multi-parameter time-varying loading. For most realistic reliability problems the load process space is of low order. As a result, the required multidimensional integration is significantly simplified. The proposed approach also has well defined steps. As a result, there is increased transparency and reduced problems of integration instability and non-convergence. Both loads and resistances are described in terms of random variable parameters and time dependent structural resistances can be considered. Numerical examples are given to illustrate the proposed method. Example applications are given for a fixed base rigid-plastic portal frame subjected to time dependent loads and resistances. Linear and non-linear limit state equations and Normal and non-Normal distribution of the random variables are considered and compared, in some cases, to the results evaluated using Monte Carlo simulation.

1. Introduction

In structural reliability analysis many structural problems are time-dependent, usually because they are subjected to loads and/or resistances that may change with time. The estimation of the time-dependent failure probability under time varying loads and resistances is of particular interest because the additional failure probability accumulated over time usually is much greater than that when the structure was first commissioned. The computational difficulties are compounded due to the randomness of the loads as well as the resistances.

One way to handle a time varying load is to include an auxiliary random variable in time-invariant second moment reliability analysis to represent a maximum (or minimum), and then use the accepted methods for time-invariant reliability analysis [1]. For linear elastic structures where the principle of superposition is valid, a simulation technique can be used to obtain the life time maximum load effect. If all the loading processes can be approximated by sparse Poisson pulses, the 'load-coincidence approach' may be used to approximate the load effect and hence the rate of failure (or outcrossing rate) using estimates of failure probabilities obtained from time-invariant methods [2].

As described previously, it is possible to formulate the time-dependent reliability problem in the load process space [3–6]. In this formulation, the problem is defined within a coordinate system for which each axis corresponds one to one to each of the load processes acting on the structure. The load process space approach has the advantage that the problem is reduced to a small number of random variables (or random processes) in the load space. The determination of the probability of failure using the conventional form of this approach is reviewed below. This is followed by a description of the proposed procedure and then by some examples to illustrate the new procedure.

2. Structural reliability formulation in the load process space

Consider an m-dimensional load process space consisting of the loads $\mathbf{q}$ acting on the structure. The loads may be represented at any point in time by the vector $\mathbf{Q}(t)$. Fig. 1 shows a two-dimensional example. Typically, the space $\mathbf{q}$ will be of low order (i.e. m is small) for most civil engineering structures since typically there are only a few load components acting on a structure. Following convention, throughout capitals denote random quantities, lower case deterministic quantities. Also, in the following context, "time-variant" refers to the case where the system is subject to one or more random processes such as stochastic loads, whereas "time-dependent" refers to situations

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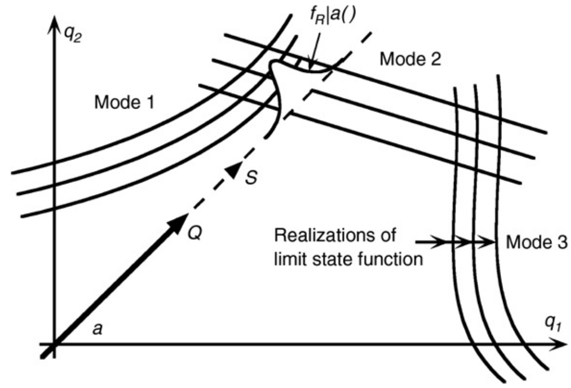

Fig. 1. Two-dimensional load-process space showing realizations of the probabilistic description of the limit state function.

where the system is changing with time such as due to mechanisms such as deterioration. [The distinction is also referred to as "fast" and "slow" time aspects respectively.]

The resistance **R** of the structure to loading may be different for various load combinations, as indicated schematically with Modes 1–3 in Fig. 1. Usually the structural resistance is composed of member resistances and other factors. Let these be represented by the random vector **X**. Then it follows that $\mathbf{R} = \mathbf{R}(\mathbf{X})$. Further, each component of **R** corresponds directly to the relevant component of **q**. Since **X** is a random vector, **R** will be random also, with joint probability density function (pdf) $f_{\mathbf{R}}()$. Further, the customary "limit state function" $G(\mathbf{x}) = 0$ in structural reliability theory is now equivalent to a probabilistic boundary in the load process space, shown as "realization of limit states" in Fig. 1. These realizations also show that the structural resistance possesses uncertainty (or variability) when plotted in the load process space.

It follows directly from the representation shown in Fig. 1 that for operations in the load process space a natural choice is polar coordinates. With this choice the probability of structural failure may be given as an integral over the radial directions $\mathbf{A} = \mathbf{a}$ [5,6]:

$$p_f = \int_{\text{unit sphere}} f_A(a) \left[\int_S p_f(s|a).f_{S|A}(s|a)\mathrm{d}s \right] \mathrm{d}a \tag{1}$$

where S is a (scalar) radial distance representing the (conditional) structural strength in the radial direction, with conditional probability density function $f_{S|A}()$. The term $p_f(s|a)$ is a conditional failure probability given that the structural resistance is $S = s > 0$ and **A** is a random unit vector of direction cosines, having a probability density function given by $f_A()$. The relationship between **R** and S is given by $\mathbf{R} = S.\mathbf{A} + \mathbf{c}$ where **c** is the point selected as the origin. For convenience of exposition, but not generally, **c** may be taken as the origin. In the exposition here, to avoid unnecessary complexity in writing the equations, time dependence is taken as implicit.

To make Eq. (1) operational the conditional probability of failure $p_f(s|a)$ over the period $(0, t_L)$ is required to be estimated for each radial direction. This can be approximated from the outcrossing rate ν and the initial failure probability $p_f(0)$ with the use of the conventional upper bound based on the Poisson nature of the outcrossings for high reliability systems [7]:

$$\begin{aligned} p_f(s|a) &\le p_f(0, s|a) + \{1 - \exp[-\nu(s|a).t_L]\} \\ &\approx p_f(0, s|a) + \nu.t_L \end{aligned} \tag{2}$$

where $p_f(0, s|a)$ is the failure probability at time $t = 0$, ν is the rate at which the vector process $\mathbf{Q}(t)$ "crosses-out" (i.e. leaves) of the safe domain and t_L is the design life. Note that $p_f(s|a)$ is a function of the distance s along the radial direction.

The other term required to make Eq. (1) operational is the variation of structural strength $f_{S|A}()$, also a function of the distance s along the radial direction. Most previous research efforts to find appropriate expressions for the variation of structural strength $f_{S|A}()$ have required significant simplifications, including [5]:

1. The limit state functions are assumed independent of the realizations of the load processes (this assumption also applies to most alternative methods)
2. The outcrossing rate ν is assumed to be independent of t_L, i.e. in Eq. (2) there is no correlation between the items in the [] term for each **a** (the so-called "ensemble" approximation).

Since structural systems usually are high reliability systems, an outcrossing event typically is a very rare event and for this situation neither of these simplifications is particularly important. However, for item 2 it has been shown that the approximation becomes worse as the uncertainty in **R** increases and as **R** becomes small relative to **Q** [8,9]. Typically, this will be the case as a structure deteriorates. Hence assumption (2) above becomes less plausible and needs attention, particularly for deteriorating structures.

3. Proposed new formulation

The approach considered herein is to revert to a more fundamental form of Eq. (1), with integration over the resistance **R** kept until the probability for a given realisation of **R** has been estimated. The approach is to consider different realizations of the structural limit state function(s) and then to determine levels of structural resistance that have equal probabilities of occurrence. It is convenient to view the sets of realizations as forming "contours" of **R**, each of equal probability and each expressed through a value s of the radial scalar S. The probability of limit state violation (i.e. failure), weighted over all possible "probabilities" $f_S(s)$ of occurrence of the limit states, may then be expressed as

$$p_f = \int_S \left[G(s) = 0 \int p_f(a|s) f_A(a)\mathrm{d}a \right] f_S(s)\mathrm{d}s \tag{3}$$

where the term $G(s) = 0$ represents the (critical) limit state function (i.e. the contour) corresponding to $S = s$. In the [] term, $p_f(a|s)$ is obtained from Eq. (2) with the different argument simply reflecting the order of integration. The term $f_A()$ is as before, representing the probability density function for the random unit vector of direction cosines **A**.

Expression (3) has the theoretical advantage that time dependence of **R**, or structural deterioration, is now governed by s and $G(s) = 0$ and the location of its contours in **q** space. Evidently, it now becomes clear that structural deterioration (or structural enhancement) will affect the location of the contours of $G(s) = 0$ in **q** space.

For structural deterioration the general tendency is for **R** and hence the contours to move closer to the origin with the passage of time. The deterioration process generally will have a degree of randomness about it, and for spatially varying systems that randomness will be different in different parts of the structure. The approach proposed herein is that this can be accounted for by considering different points in time at which the deterioration is estimated (as a random variable) and hence the structural capacity is estimated. It is assumed that the structural deterioration process is smooth in time and there are no 'shocks' or discontinuities. Hence, the precise location of the estimation points in time is not critical. It follows that (3) also should be written as conditional on time (i.e. it is time-dependent):

$$p_f|t = \int_S \left[G(s,t) = 0 \int p_f(a|s) f_A(a)\mathrm{d}a \right] f_S(s)\mathrm{d}s \tag{4}$$