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Lattice flexures: Geometries for stiffness reduction of blade flexures

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ABSTRACT

Reducing the motion-direction stiffness of compliant mechanisms reduces their actuation effort and simplifies associated static balancing mechanisms. This work introduces a flexure type called lattice flexures and evaluates some of their fundamental properties. Lattice flexures have a reduced bending stiffness when compared to traditional rectangular-section blade flexures of similar size. The motion-direction bending stiffness of two lattice flexure types, called X-type and V-type, are analytically derived, corroborated with finite element analysis, and validated with measurements of physical prototypes. The lattice flexure has the potential to reduce the bending stiffness of some compliant mechanisms by 60–80%, as demonstrated in devices manufactured using 3D printing technologies. It is shown that some lattice flexures exhibit a torsional/bending stiffness ratio as much as 1.7 times higher than an equal aspect-ratio blade flexure, and a transverse bending/motion-direction bending stiffness ratio up to 6.5 times higher than an equal aspect-ratio blade flexure.

1. Introduction

This paper introduces the lattice flexure as a means to reduce the motion-direction rotational stiffness of compliant mechanisms. A compliant mechanism obtains its motion from the deflection of its constituent members. This eliminates sliding contact of surfaces, avoiding friction and subsequent wear, and leading to higher performance [1]. Because of the strain energy associated with bending the flexible members, compliant mechanisms often have higher actuation effort compared to traditional mechanisms [2]. Static balancing is one way of reducing this actuation energy [3–8]. Static balancing functions by introducing balancing elements that store and release energy as the mechanism is actuated. Because the net change in energy stored by the mechanism is small, the actuation effort is reduced. However, stiffer mechanisms require that more strain energy be stored by the balancing element [9]. By reducing the initial mechanism stiffness, simpler balancing elements can be used.

The stiffness of a flexure is governed by its material, boundary conditions, and geometry [1]. This work will consider the stiffness of an arbitrary material with elastic linear stiffness (i.e. constant Young's modulus in the elastic range). The boundary conditions are that of a cantilever beam subject to an end moment load. Therefore, the aspect of beam stiffness to be examined is beam geometry.

The conventional blade (or leaf-spring) flexure design is that of a prismatic rectangular-section beam [10] shown in Fig. 2a. Much work has been done studying this kind of flexure to gain greater insight into its non-linear deflection and stiffness [11]. Changing the beam length, width, or thickness will result in a change in stiffness. The bending stiffness of a cantilever beam subject to a moment load is $k_b = EI/L$ [12]. E is the Young's Modulus, I is the second moment of area, which for rectangular sections is given by $bh^3/12$. L, b, and h are the length, width, and thickness of the flexure, respectively.

Decreasing a flexure's thickness can be a straightforward and efficient way of decreasing the bending stiffness. The lower bound of thickness is generally dictated by the available materials (stock sizes) and manufacturing processes or other design constraints. For example, in 3D printing, a process such as electron beam melting may be able to reliably print features no smaller than 1.0 mm thick [13]. Flexure width is limited in a similar way, with the addition that the flexure stability (its ability to withstand off-axis loads) decreases as the width decreases. Flexure length is often limited by mechanism envelope. Thus a flexure designer may arrive at the practical geometric limits of a flexure but still be unsatisfied with its performance [14]. The lattice flexure is introduced as one way of addressing this issue. This can be important in applications where it is necessary to reduce actuation effort while maintaining comparable stiffness in off-axis directions (such as in space applications where actuation effort can be proportional to actuator size, which can be proportional to actuator mass).

Flexures are important elements in many mechanical systems [15–17]. Different types of flexures have been the focus of recent

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Fig. 1. A 3D printed titanium cross-axis flexural pivot with an early lattice flexure design.

studies, including cross-axis flexure pivots [18], cartwheel flexures [19], trapezoidal flexures [20], and others [21]. Methods for modeling and design of flexures include the pseudo-rigid-body model [22,23], FACT [24], screw theory [25], matrix methods [26], and analytic methods paired with finite element analysis [10]. These methods differ in accuracy and complexity, but all are meant to aid the designer in arriving at a suitable configuration of flexures. Common concerns in flexure design include stiffness [17], stress and fatigue life [1], and off-axis (non motion-direction) stiffness [19].

In this work we introduce the lattice flexure, a new flexure type that has an envelope similar to a blade flexure but has dramatically reduced motion-direction bending stiffness and an increased ratio of support-direction to motion-direction bending stiffness. This reduced stiffness lowers the required actuation effort and simplifies the design of any static balancing system incorporating a lattice flexure. Lattice flexures have significantly lower mass while maintaining good off-axis stiffness. Fig. 1 shows an early lattice flexure design in a 3D-printed titanium cross-axis flexural pivot. While this introductory paper cannot exhaustively investigate every aspect of lattice flexure behavior, some investigation of its bending and torsional stiffness properties is presented. The improved performance is countered by increased manufacturing complexity. Advances in additive manufacturing and make monolithic fabrication of such flexures feasible.

2. Approach

Fig. 2 shows a conventional blade flexure and the proposed geometry for two lattice flexure types. Fig. 2b shows the geometry of an X-type flexure, so named for the crossing of the diagonal lattice elements. Fig. 2c shows the geometry of a V-type lattice flexure, so named because of the diagonal elements' resemblance to the letter "V." Both the X-type and V-type flexures are characterized by the ratio L_1/b and the aspect ratio η, where $\eta = h/(h+b)$ (the thickness over the overall width). By removing material from the middle of the blade flexure the effective width is reduced. This reduces stiffness, and because the diagonal elements of the lattice are in combined bending and torsion (rather than pure bending), the percent reduction in bending motion stiffness is greater than the percent of material removed. While many geometries incorporating these concepts can be conceived, the geometries herein presented are meant to be a proof-of-concept and a starting point for further development.

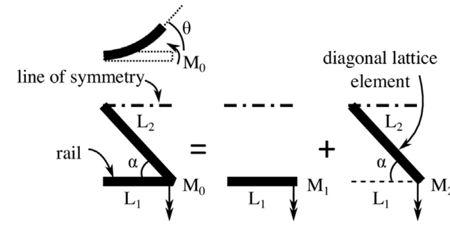


Fig. 3. The X-type flexure is analyzed using the symmetry about the central plane of the flexure, using the variables L_1, L_2, and α.

2.1. Stiffness of lattice flexures

In this section we derive the stiffness of X-type and V-type lattice flexures. Fig. 3 shows a single geometric unit of a lattice flexure and the nomenclature used in the derivation.

First we will derive the stiffness of an X-type lattice flexure. Variables used in this derivation are depicted in Figs. 2 and 3. Note that the stiffness of only one quadrant of the X is analyzed and the overall stiffness is found from symmetry. The bending moment M_0 applied to the lattice element can be decomposed into the moments M_1 and M_2. M_1 is the moment carried by the "rails" of the lattice, while M_2 is the moment carried by the diagonal lattice element. The deflected angle θ induced by M_1 can be found from elementary beam theory as

$$\theta = \frac{M_1 L_1}{E I_r} \tag{1}$$

where I_r is the second moment of area of the rail, E is the Young's modulus, and L_1 is the length of one half-unit cell of the X-type flexure.

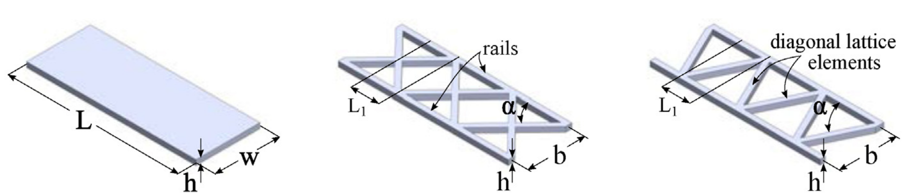
Because the ends of the two segments are rigidly joined, the diagonal lattice element must also be deflected to this angle (θ). The angular deflection of the diagonal lattice element due to torsion (θ_t) in the beam is given by

$$\theta_t = \frac{M_2 \sin(\alpha) L_2}{KG} \tag{2}$$

where L_2 is the length of the diagonal lattice element, M_2 is the component of the applied moment (M_0) reacted by the diagonal lattice element, K is a section property (a function of lattice element cross-section [12]), G is the modulus of rigidity, and α is the lattice angle (see Figs. Figs. 2b and 3). The angular deflection due to bending (θ_b) is given by

$$\theta_b = \frac{M_2 \cos(\alpha) L_2}{E I_l} \tag{3}$$

where I_l is the second moment of area of the diagonal lattice element.



(a) A conventional blade flexure. (b) An X-type lattice flexure. (c) A V-type lattice flexure.

Fig. 2. Examples of different flexure types. Note that b is the distance between the rail centers, not the full width of the flexure.