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Mechanistic force model coefficients: A comparison of linear regression and nonlinear optimization

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ABSTRACT

This paper describes a comparative study devised to examine the dependence of specific force coefficients, which are used in mechanistic cutting force models, on various milling process parameters such as feed per tooth, spindle speed, milling configuration, and radial immersion. Two methods are described for determining the specific force coefficients: (1) the average force, linear regression method; and (2) the instantaneous force, nonlinear optimization method. A series of test cuts were performed and the specific force coefficients calculated using the two methods were compared. Additionally, a technique for extending the bandwidth over which the cutting forces were measured using a commercially available cutting force dynamometer is presented. Finally, a series of milling stability experiments were conducted to validate the calculated specific force coefficients. It was found that milling process parameters such as feed per tooth, spindle speed, and radial immersion exhibit a nonlinear relationship with the specific force coefficients.

1. Introduction

The modeling of machining processes, which has been an important research topic for nearly a century, is motivated by the requirements of both machine tool users and builders. The machine tool user aims to reliably predict key process outputs, including cutting forces which affect workpiece surface quality, workpiece geometric accuracy, and process stability. From the builder's perspective, the cutting forces represent a critical design metric because they dictate the required spindle power and torque as well as the required rigidity of the machine tool's structural loop. In machining process simulations and optimizations, cutting force modeling strongly affects the accuracy of the results.

There are three approaches to cutting force modeling which are described in the literature: analytical, numerical, and mechanistic [1,2]. The analytical models relate cutting forces to a number of process variables (i.e., feed per tooth, cutting speed, and cut geometry) and mechanical aspects such as shear angle, material properties, and friction. Early work using this approach was detailed by Merchant in [3] and by Amarego and Brown in [4]. Increased computational power has led to advancements in numerical modeling, where the relationship between chip and tool geometry is studied [5].

The mechanistic force models assume that the instantaneous cutting forces are proportional to the uncut chip area through one or more empirical coefficients [1]. Early work in mechanistic force modeling for milling operations was reported by Martellotti [6], Koenigsberger et al. [7], and Sabberwal [8]. The literature highlights two primary mechanistic force models. The first relates instantaneous cutting forces and uncut chip areas to a single empirical coefficient which is commonly referred to as the specific (cutting) force coefficient, K_s. This coefficient captures the effect of both cutting (shearing) and ploughing (due to friction at the cutting edge) which occurs during chip formation. The ease of implementation and predictive capabilities provided by this simple model have resulted in its widespread application in industry and research. The second, published in later works by Budak et al. [9], extends the mechanistic cutting force model to include separate empirical coefficients to capture the chip formation mechanics of shearing and ploughing.

In [10,11], a method for the identification of the empirical coefficients, commonly referred to as specific force coefficients, is presented. The procedure proposes that a linear regression of measured cutting forces be performed over a range of feed per tooth values while holding other process parameters such as cutting speed and cut geometry constant. This method, which requires several cutting tests to perform the linear regression analysis, provides

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results which are specific to the selected cutting tool geometry and workpiece material combination. The regression analysis assumes that the cutting forces are linearly dependent on feed per tooth and independent of other machining parameters such as cutting speed and feed, cut geometry, and cut direction (i.e., up milling/down milling). Other methods, such as those presented in [12,13], use nonlinear optimization methods to perform a least squares fit of simulated cutting forces to measured cutting forces. This approach requires measurements from a single cutting test and, again, results in specific force coefficients which are specific to the chosen machining parameters. As such, the specific force coefficients may be considered to be a function of not only the cutting tool geometry and workpiece material, but also machining parameters, such as cutting speed, feed, cut geometry, and cut direction. The nonlinear optimization method provides a tool for studying the effects of these machining parameters on dynamic cutting forces.

It has been demonstrated in the literature that the measurement of cutting forces with piezoelectric dynamometers at high cutting speeds is complicated by the dynamic influence of the instrument. As the tooth passing frequency and subsequent harmonics exceed the dynamometer bandwidth (i.e., the frequency range over which the frequency response is nominally constant) spurious frequency content is superimposed on the measured cutting forces. In [14] Altintas et al. use a Kalman filtering approach to remove the dynamic influence of a novel spindle-integrated force sensor's structural modes. Other research, such as that presented in [15–24], has also addressed the challenges associated with limited dynamometer bandwidth using force-to-force frequency response function (FRF) measurement, filtering, and other techniques.

In this paper, a comparative study is described where the specific force coefficients of the mechanistic force model are determined using both linear regression and nonlinear optimization methods. The paper is organized as follows. First, the mechanistic force model is detailed, and the two methods for specific force coefficient determination are described. Next, the experimental method, which includes the cutting force measurement, the dynamic compensation technique, and the stability testing setup, is detailed. Finally, the resultant cutting force coefficients are compared and the practicality of the nonlinear optimization method is demonstrated in the framework of a milling stability prediction via time domain simulation and experimental validation. This is followed by a discussion of the experiment results and their impact on finish milling operations at low radial immersion.

2. Mechanistic force modeling

The mechanistic force models are based on the assumptions that: (1) the instantaneous cutting force is proportional to the cross sectional area of the uncut chip through empirical specific force coefficients; and (2) the instantaneous cutting forces are independent of other machining parameters. Although these assumptions provide a reasonable degree of accuracy for milling stability prediction using stability lobe diagrams [10], it has been shown that cutting forces are dependent on cutting speed and feed [13]. The mechanistic force model used in this study includes instantaneous cutting forces in the tangential, F_t, normal, F_n, and axial, F_a, directions with six corresponding force coefficients; see Eqs. (1)-(3):

$$F_t = k_{tc}bh + k_{te}b \tag{1}$$

$$F_n = k_{nc}bh + k_{ne}b \tag{2}$$

$$F_a = k_{ac}bh + k_{ae}b \tag{3}$$

where b is the chip width (i.e., axial depth of cut in milling) and h is the instantaneous chip thickness, which is based on the circular tooth path approximation; see Eq. (4):

$$h = f_t \ \sin(\phi) \tag{4}$$

where f_t is the feed per tooth and ϕ is the cutter rotation angle. Each component of the instantaneous cutting force includes two specific force coefficients. The coefficients k_{tc}, k_{nc}, and k_{ac} are correlated with cutting and the edge coefficients k_{te}, k_{ne}, and k_{ae} are correlated with ploughing. The edge coefficients affect the instantaneous cutting force proportionally through the chip width, but are independent of the instantaneous chip thickness.

2.1. Average force, linear regression method

The six specific force coefficients were determined through linear regression analysis using the average cutting forces measured during a series of cutting tests which were performed over a range of feed per tooth values while holding other milling parameters constant. Projecting the tangential, normal, and axial cutting force components into a fixed reference frame (i.e., x, y, and z), shown in Fig. 1, and averaging over one cutter revolution yields the following expressions for mean cutting force per revolution.

$$\bar{F}_x = \left\{ \frac{N_t b f_t}{8\pi}[-k_{tc}\cos(2\phi) + k_{nc}(2\phi - \sin(2\phi))] + \frac{N_t b}{2\pi}[k_{te}\sin(\phi) - k_{ne}\cos(\phi)] \right\}_{\phi_s}^{\phi_e} \tag{5}$$

$$\bar{F}_y = \left\{ \frac{N_t b f_t}{8\pi}[k_{tc}(2\phi - \sin(2\phi)) + k_{nc}\cos(2\phi)] - \frac{N_t b}{2\pi}[k_{te}\cos(\phi) + k_{ne}\sin(\phi)] \right\}_{\phi_s}^{\phi_e} \tag{6}$$

$$\bar{F}_z = \left\{ \frac{N_t b}{2\pi}[k_{ac} f_t \cos(\phi) - k_{ae}\phi] \right\}_{\phi_s}^{\phi_e} \tag{7}$$

where N_t is the number of teeth on the cutter and ϕ_s and ϕ_e are the start and exit angles of each tooth based on the radial depth of cut and cut direction.

Often, 100% radial immersion (i.e., slotting) cutting tests are selected, where $\phi_s = 0^\circ$ and $\phi_e = 180^\circ$, so that the mean cutting force per revolution expressions simplify to:

$$\bar{F}_x = \frac{N_t b k_{nc}}{4} f_t + \frac{N_t b k_{ne}}{\pi} \tag{8}$$

$$\bar{F}_y = \frac{N_t b k_{tc}}{4} f_t + \frac{N_t b k_{te}}{\pi} \tag{9}$$

$$\bar{F}_z = -\frac{N_t b k_{ac}}{\pi} f_t - \frac{N_t b k_{ae}}{2} \tag{10}$$

These expressions are provided in slope–intercept form. A linear regression over feed per tooth may be performed to determine the cutting force coefficients. The slope, a_{1j}, and intercept, a_{0j}, of the linear regression are given as:

$$a_{1j} = \frac{n\sum_n^{i=1} f_{t,i}\bar{F}_{j,i} - \sum_n^{i=1} f_{t,i}\sum_n^{i=1}\bar{F}_{j,i}}{n\sum_n^{i=1} f_{t,i}^2 - \left(\sum_n^{i=1} f_{t,i}\right)^2} \tag{11}$$

$$a_{0j} = \frac{1}{n}\sum_{i=1}^{n}\bar{F}_{j,i} - a_{1j}\frac{1}{n}\sum_{i=1}^{n} f_{t,i} \tag{12}$$

where j indicates the force component direction (i.e., x, y, or z) and n is the number of data pairs $(f_{t,i}, \bar{F}_{j,i})$. Once the slope and intercept