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Design and nonlinear analysis of a 6-DOF compliant parallel manipulator with spatial beam flexure hinges

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ABSTRACT

In this paper, a compliant parallel manipulator with six compliant limbs is proposed for micro positioning applications. The load–displacement model of a single compliant limb is established using a nonlinear closed-form spatial beam model. The inverse solution to the compliant parallel manipulator is then implicitly derived by applying load equilibrium to the moving platform. Finally, the compliant model of the limb and the implicit inverse kinematic solution of the manipulator are fully tested by FEA. Discrepancies between results of the presented models and the FEA are analyzed within planned workspaces. The validations demonstrate that accuracies of the proposed models are acceptable and can be improved by shrinking the planned workspace.

1. Introduction

Compliant parallel manipulators (CPMs) provide predetermined motion due to the elastic deflections of flexure modules and can be utilized in various applications [1]. Compared with the conventional parallel mechanism, a CPM is well known for its potential merits, such as zero backlash, high precision, lack of friction and wear. Due to these advantages, CPMs have been widely studied [2–4] and become more and more popular in the parallel mechanism research community.

One of the most commonly used flexure hinges is the spatial beam flexure hinges (SBFH) [5–7]. The SBFH, which is usually referred as a uniform, symmetric cross-section, slender and spatial beam flexure, has three relative high compliances, i.e. two bending compliances and one-torsion compliance about its centroid axis. On the other hand, the translational compliance along the centroid axis is much lower. Therefore, a SBFH generally has five degrees of freedom (DOF) as well as one degree of constraint (DOC) [8] and is usually treat as a compliant spherical joint [1,4,8].

In previous studies of CPMs, finite element analysis (FEA) based methods are still prevailing in solving the load–displacement relations [1,3,9]. In these studies, basic components of the CPMs, such as flexure hinges and struts, are generally analyzed using beam elements based on finite element method (FEM) where nodal stiffness matrices are first derived and then assembled to establish the stiffness matrix of the whole manipulator. In practice, stiffness matrix of the CPM can be linearly formulated, but it is only accurate within a very small workspace [10]. Although FEA method usually has a neat mathematical form and relative high efficiency, parametric design insights which are strongly demanded in the design stage become ambiguous.

In order to find a tradeoff between accuracy and mathematical complexity, while taking into account the demands of parametric design, several beam formulations, which are capable of capturing all the measurable deformations of the SBFH, are developed. Hodges and Dwell's work [9] presents a group of slender beam formulations considering cross-sectional distortion and warping along with combined bending, torsional and axial loads for dynamic analysis of a helicopter rotor blade. Their formulations are approximated to the second and third order, but the explicit load–displacement relations are not provided in either analytical or closed-form. An analytical model has been presented for a three-legged table with vertical beam flexures in Hao's work [11], but this model fails to capture the coupling between the two bending directions. Moreover, the analytically deduced model has difficulty in dealing with flexure mechanisms with four or more legs because of the mathematical complexity. Recently, Sen and Awtar's work provides a closed-form nonlinear model to formulate the load–displacement relations of a multi-legged table flexure mechanism [6]. The beam model can provide a closed-form solution to the SBFH with 95% accuracy over a range of ± 0.1 L ($\sim$10% of the beam length) and ± 0.1 rad for translational and rotational displacements, respectively. However, the principle of virtual work (PVW) based method that was adopted in Ref. [7] can hardly be applied to

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the CPM with multi-segmented limbs, such as PSS limbs referred in Ref. [3].

The motivation of this paper is to develop a novel inverse kinematic model for our newly designed 6-DOF CPM with six three-segmented identical compliant limbs. The load–displacement model of the three-segmented compliant limb and the inverse kinematics model to the 6-DOF CPM will be established in closed-form. Corresponding validations will be performed in interested workspaces in order to prove the accuracy of the presented models. Comparing with the existed models, the proposed model can be solved without the assistance of FEA based methods and will provide more parametric design insights.

The reminder of this paper is organized as follow. The structure of the proposed 6-DOF CPM is described in Section 2, then the load–displacement model of a single compliant limb is derived in Section 3. Section 4 presents the inverse kinematic model to the designed 6-DOF CPM. All of the proposed models are then validated with FEA in Section 5. At last, conclusions will be given in Section 6.

2. Conceptual design of a 6-DOF parallel manipulator

The prototype of the proposed 6-DOF CPM is depicted in Fig. 1 and the design parameters of the CPM are listed in Table 1. Detailed structural descriptions are demonstrated in Fig. 2. It is shown that the designed parallel manipulator using SBFH at all joints is composed of a moving platform, six identical limbs and six vertical linear actuators fixed to the base platform. The six linear actuators are symmetrically arranged about the $\boldsymbol{x}_0$-axis of the coordinate system $O[\boldsymbol{x}_o, \boldsymbol{y}_o, \boldsymbol{z}_o]$ at the center of the fixed base. Each of the six

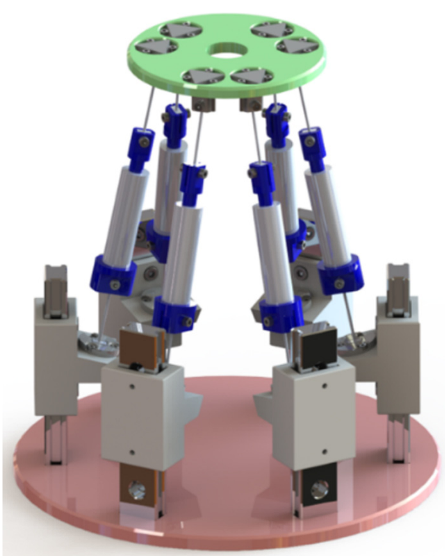

Fig. 1. The prototype of the 6-DOF CPM.

Table 1
Geometric and material parameters of the 6-PSS prototype.

Item	Value
Distribution radius of upper SBFH: r	40 mm
Distribution radius of lower SBFH: R	80 mm
Distribution angle of upper SBFH: γ	30°
Distribution angle of lower SBFH: κ	60°
Radius of the SBFH: r_f	1 mm
Length of the SBFH: L_f	40 mm
Radius of the strut: r_s	10 mm
Length of the strut: L_s	100 mm
Modulus of elasticity of SBFH: E_f	113 GPa
Modulus of elasticity of the limb: E_s	206 GPa

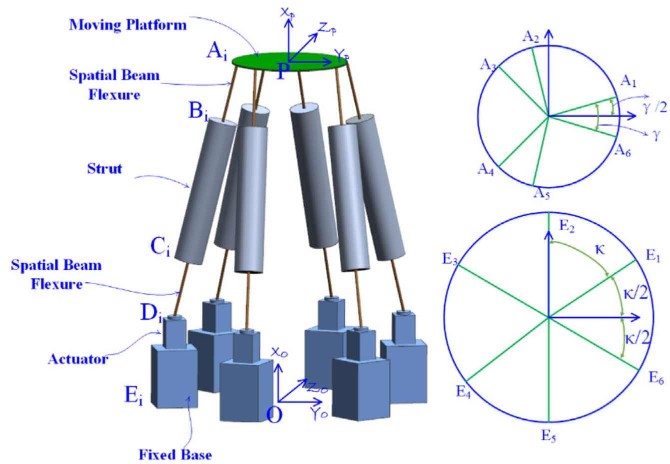

Fig. 2. Structural description of the spatial 6-PSS parallel manipulator.

identical limbs connects the moving platform to the fixed base with an upper SBFH followed a rigid strut and a lower SBFH in sequence.

A moving frame $\mathbf{P}[\boldsymbol{x}_P, \boldsymbol{y}_P, \boldsymbol{z}_P]$ is settled at the center of the moving platform. Corresponding component vectors of the frame **O** and the frame **P** are parallel to each other at the initial configuration where the principal planes of the fixed base and moving platform are parallel and the line **OP** is perpendicular to the fixed base plate. Point $\mathbf{E}_i$ (i = 1 to 6) denotes the ith immobile reference point on the fixed base while point $\mathbf{A}_i$ (i = 1 to 6) indicates the intersection point where the ith SBFH joins the moving platform. The layout of the points $\mathbf{A}_i$ and $\mathbf{E}_i$ are presented in Fig. 2 while the related parameters are listed in Table 1.

In the current design, all the actuators are fitted on the fixed base, which makes the six identical limbs free of external disturbances that induced by electrical wires of actuators. Furthermore, the proposed CPM can be designed as either a micrometer or a nanometer positioning platform and hence the actuators should be chosen according to the design objective.

To avoid the monolithic feature [3] and achieve a large workspace, the SBFH is employed as the compliant spherical joint in this paper. The load–displacement relations to the utilized SBFH can be expressed in a closed-form model under the assumption of Euler–Bernoulli beam theory [12]. Furthermore, to maximize the workspace of the designed parallel platform, the chosen materials should allow great elastic deformations before yield, hence the materials with the greatest ratios between the yield strength and the Young's modulus (σ/E) are preferred. The Ti–Al–4V alloy with relative high Young's modulus (113 GPa) and σ/E (0.0078) is selected as the material for the SBFH.

3. Load–displacement model of the compliant limb

3.1. Load–displacement model of the SBFH

The SBFH adopted in this paper is a slender beam flexure with circular cross-section. As shown in Fig. 3, the SBFH is uniform along its length subject to a general end-loading and point $\mathbf{Q}_\mathrm{d}$ = [X_0, 0, 0] is the center of an arbitrary cross-section that is at distance X_0 from the beam-root and perpendicular to the centroidal axis prior to deformation. The translations and rotations of the cross-section $\mathbf{Q}_\mathrm{d}$, which are caused by a given end load [F_{XL}, F_{YL}, F_{ZL}, M_{XL}, M_{YL}, M_{ZL}], are given as $\Delta\mathbf{Q}$ = [u, v, w] and ψ = [α, β, θ_d] in Fig. 3.

According to Euler's deformation assumptions, the cross-section $\mathbf{Q}_\mathrm{d}$ remains plane and perpendicular to deformed centroidal axis. In deformation, the entire cross-section denoted by **Q** translates as a rigid plane from **Q** to $\mathbf{Q}_\mathrm{d}$ = [$X_0 + u$, v, w] by undertaking three