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Review

Necessary and sufficient mobility conditions for single-loop overconstrained *n*H mechanisms

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ABSTRACT

For overconstrained single-loop mechanisms with helical joints (*n*H mechanisms) necessary conditions for the finite mobility are derived using a local approximation of the loop closure conditions by means of a TAYLOR-series expansion of the 1*st*-order closure conditions. The loop closure conditions and their derivatives are formulated in terms of the screw coordinates of the helical joint axes, whereby the series terms are recursively expressed. Although it has been shown by means of the ARTIN approximation theorem that there exists a finite local approximation order $m_{\max}$ that guarantees finite mobility of the mechanism, the value of this approximation order is in general unknown. For a numerical estimation for m_{max} an algorithm for the solution of the m nonlinear mobility conditions has been developed to predict the sufficient local approximation order m_{num} for various *n*H mechanisms. The procedure enables the synthesis of overconstrained *n*H mechanisms.

Contents

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1. Introduction

Overconstrained mechanisms have full cycle mobility although they have zero or less degrees of freedom according to the formula of GRÜBLER–KUTZBACH–CHEBYCHEV. This property is caused by the dependency of the loop closure conditions due to special geometric properties. The investigation of overconstrained mechanisms has been a topic in kinematics since a long time.

Tasks for the analysis of overconstrained mechanisms are the formulation of necessary and sufficient conditions and a prediction of the mobility as well as the analysis of the input–output equations in the parameter space and of the movement of the mechanism links in the configuration space. The synthesis comprises the multi-pose synthesis for known overconstrained mechanisms and the topological synthesis using necessary and sufficient conditions for the mobility to find new overconstrained mechanisms. For both tasks, analysis and synthesis, various mathematical methods have been developed until now. Starting point for the most of these methods are the implicitly formulated loop closure conditions at the position level. For overconstrained mechanisms with prismatic and revolute joints these closure conditions can be algebraically formulated, and approaches like GRÖBNER basis combined with kinematic mapping can be used [1–4]. For overconstrained mechanisms with helical joints methods were developed from the view point of the geometrical interpretable screw theory [5–8] and the more general group theory [9–11] as well as algebraic methods [12]. Since an explicit solution of the closure conditions is impossible in general, a higher-order local approximation of the implicitly formulated closure conditions can be used for the prediction of the mobility [13–16].

The present contribution describes a new formulation of sufficient and necessary conditions for the mobility of single-loop overconstrained mechanisms with helical joints (nH mechanisms), expressed by the actual poses of the joint axes. The principle is to formulate higher-order local approximations of the implicitly formulated 1*st*-order closure conditions in an actual position of the mechanism yielding an infinite number of necessary closure conditions for the mobility. Based on screw theory the derivatives of the closure conditions can be algebraically expressed using the principle of transference. The relative joint angles and their derivatives are eliminated from the closure conditions by taking the screw system being reciprocal to the screw axes of the mechanism into account. Thus the obtained necessary mobility conditions depend on the screw coordinates of the joint screws only. Using the ARTIN approximation theorem, it can be shown that there exists an unknown sufficient local approximation order which guarantees the finite mobility of the nH mechanism [14].

The paper is organised as follows. In Section 2 the 1*st*-order closure conditions of general single-loop mechanisms with helical joints are formulated. In Section 3 necessary conditions for the mobility of overconstrained mechanisms are formulated by means of a series-expansion of the 1*st*-order closure conditions. In Section 4 overconstrained single-loop mechanisms with helical joints are synthesised by solving the mobility conditions. At first the number of unknowns is further reduced by taking analytical solutions of the 1*st*- and 2*nd*-order mobility conditions into account. Two special cases are shown for which the fulfilment of the 1*st*- and 2*nd*-order mobility conditions is already sufficient for the fulfilment of all higher-order mobility conditions. Subsequently an algorithm is presented that numerically delivers a finite order of the mobility conditions that is sufficient for the finite mobility of the nH mechanism. The formulations of the mobility conditions are based on screw theory, whereby the *motor* notation is used that is briefly summarized in the appendix.

2. First-order kinematics of a *n*H mechanism

For a general single-loop mechanism with n helical joints (nH mechanism) and bodies according to Fig. 1 the 1*st*-order closure conditions are formulated for the subsequent mobility analysis. For this purpose the loop in Fig. 1 is opened by duplicating body n into bodies 0 and n yielding an open nH chain with base body 0 and end body n.

2.1. First-order kinematics of an open nH chain

The relative screw motions of the bodies of the open nH chain are described by the relative helical joint angles $q_i, i = 1, \ldots, n$ around the screw axes of the helical joints that are expressed in terms of normalised screw coordinates written in the motor representation, refer to the Appendix,

$$\hat{\boldsymbol{a}}_i \equiv \begin{bmatrix} \boldsymbol{a}_i \\ \boldsymbol{a}_{\varepsilon i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_i \\ \widetilde{\boldsymbol{r}}_i \boldsymbol{a}_i + h_i \boldsymbol{a}_i \end{bmatrix}, \quad i = 1, \ldots, n, \tag{1}$$