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The best-approximate realization of a spatial stiffness matrix with simple springs connected in parallel

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This paper presents an approximation algorithm to simplify the physical implementation of the non-isotropic stiffness matrices. The algorithm is inspired by the best approximation theorem in Matrix theory, by which the original stiffness matrix is projected onto a special linear space, thus an approximate isotropic matrix is derived. Accordingly, the desired force–deflection characteristics can be realized by purely simple springs (linear/torsional springs) connected in parallel. In mathematics, the approximation only changes the diagonal entries of the off-diagonal blocks of the original stiffness matrix. Moreover, the physical properties of the approximate isotropic matrix are identical to that of the original stiffness matrix in many aspects, including the center of stiffness, the wrench compliance axes as well as the translational and rotational stiffness value. The approximation is unique in both mathematics and physics. In order to illustrate the effectiveness of the proposed method, a numerical example is studied with comparison to some traditional algorithms of stiffness decomposition. The advantages of the approximation are well verified through the example.

1. Introduction

Appropriate mechanical interaction between the manipulator and circumstances is one of the most important problems in the use of robots in manufacturing tasks such as material removal and automated assembly. Without some form of force regulation, excessive interaction forces may arise from small manipulator positional misalignment and then lead to the failure of task [1]. To avoid damage to both the manipulator and the environments, the passive compliance which can be modeled as a kinematically unconstrained rigid-body suspended by several elastic units is generally used to provide some degrees of compliance at the end effector of a robot. The typical example of such a device is the Remote Center Compliance (RCC) device [2]. Proper design and use of such devices can result in a significant improvement in the reliability of manufacturing tasks [3].

In mathematics, the compliance/stiffness requirements can be quantized by a 6×6 stiffness matrix. As a result, the design of passive compliance can be transformed as a decomposition of the desired stiffness matrix. The decomposition will produce several spring wrenches. The wrench determines the placement and type of the passive spring that connects the suspended rigid-body and base of the passive compliance device. Many researchers have made contributions to the synthesis and realization of desired spatial stiffness matrix. Notably, Loncaric [4] pioneered an algorithm to synthesize a realizable spatial stiffness matrix with no more than 20 linear springs. Then the number of required simple springs for synthesizing the realizable stiffness matrix was

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reduced to seven at most by Huang and Schimmels [5]. The two scholars [6] also proposed the eigenscrew decomposition which is invariant in the coordinate transformation. Lipkin and Patterson [7,8] presented the concept of wrench compliance axis and its dual, namely twist compliance axis. The former refers to a wrench along the axis yields a parallel translational deflection, and the latter implies a twist deflection along the axis causes a parallel reacting couple. The results were adopted to geometrically decompose the stiffness matrices into two sets of orthogonal spring wrenches with finite and infinite pitches [8,9]. In order to reduce the complexity of the compliance device, the minimal realization of the desired stiffness matrix was put forward by Roberts [3,10] to minimize the number of passive springs. In addition, Li [11] researched the realization of an isotropic stiffness matrix using springs selected based on their positions and directions. Hong and Choi [12] also developed an algorithm for the physical realization of any rank-r isotropic stiffness matrix.

It is easy to understand that the structure of the passive compliance device should be as simple as possible for convenience of engineering implementation. Therefore, the simple springs including the linear springs and torsional springs are preferred to realize a desired stiffness performance. It is worth mentioning that the corresponding wrenches of both the linear spring and the torsional spring are isotropic with zero and infinite pitches, respectively [7].

However, not all of the stiffness matrices can be realized by purely simple springs connected in parallel. The stiffness matrices that can be decomposed into purely isotropic wrenches are 20-dimensional rather than 21-dimensional [4]. In fact, most stiffness matrices would have to be achieved with using screw springs. There are two types of screw springs [13], both of which have nonzero and finite pitches. The structures of the screw springs are shown in Fig. 1. When a force is imposed on the connected rigid-body through the screw spring, a coupled torque must be produced synchronously. Hence, the force–deflection characteristics of the screw springs are very complicated. It is unpractical for the physical implementation of a mechanism with several screw springs connected in parallel.

Roberts [3] has tried to minimize the number of screw springs when realizing a desired stiffness matrix. However, the algorithm cannot completely eliminate the screw springs for most stiffness matrices. As the stiffness matrices are strictly quantitative in mathematics, the described force–deflection characteristics are impossible to be realized precisely for engineering implementation. Inspired by this, the best approximation to stiffness matrix is proposed in this paper. The approximation aims to turn the original complex stiffness matrix into a special approximate matrix so that the desired stiffness performance can be approximately realized by purely simple springs connected in parallel. At this time, the force–deflection characteristics change a little, but the implementation of the mechanism will be easy and practical.

The advantage comes with a price. There must be deviations between the approximate stiffness matrix and the original matrix. Thus an evaluation on the differences is necessary. The existing evaluation induces on the properties of stiffness matrix mainly focus on the eigenvalue [14], determinant [15], trace [16,17], condition number [18], and norm [19] of the matrix. Nevertheless, the physical meaning of these induces is not explicit enough for the entries of stiffness matrix are of heterogeneous units. Meanwhile, many studies concentrating on the structure and mechanical characteristics of the stiffness matrix have also been developed. Typically, Loncaric [20] proposed the normal form of stiffness matrix in which the rotational and translational aspects are maximally decoupled. The form is obtained in a special coordinate frame whose origin is termed the center of stiffness [4]. Lipkin et al. [7–9] researched the wrench and twist compliance axes which are very important for they can lend themselves to practical mechanism operations as they define the directions of simple end-effector reactions arising from contact with the surroundings. The centers of the two kinds of compliance axes were proved to be coincident and defined as the center of elasticity [21]. Moreover, El-Khasawneh and Ferreira [22] pointed out that one is usually more interested in the mechanical response of a system in the direction of excitation for many engineering problems, and in consequence, the single-dimensional stiffness defined as the ratio of the magnitude of the wrench to that of the twist is proposed. Portman [23] further put forward the concept of collinear stiffness value (CSV) that embodies the effective stiffness value in one direction. The term collinear means that an initial displacement and resulting force are measured along and about the same axes.

According to the researches above, this paper further presents a physical appreciation on the best approximation. The force–deflection characteristics of the approximate stiffness matrix in different directions are compared with that of the original stiffness matrix. This work can provide guidance for the researchers, to decide whether they can adopt the algorithm to simplify the realization of stiffness requirements.

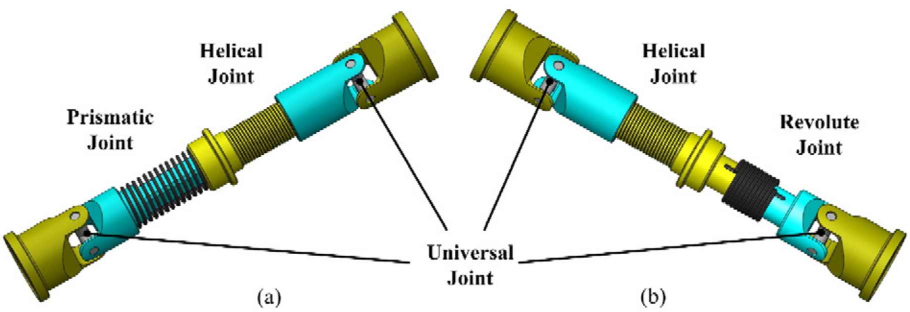

Fig. 1. Screw springs. (a) Translational type screw spring (b) Rotational type screw spring.