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A Kalman filter based strategy for linear structural system identification based on multiple static and dynamic test data

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Abstract

The problem of identification of stiffness, mass and damping properties of linear structural systems, based on multiple sets of measurement data originating from static and dynamic tests is considered. A strategy, within the framework of Kalman filter based dynamic state estimation, is proposed to tackle this problem. The static tests consists of measurement of response of the structure to slowly moving loads, and to static loads whose magnitude are varied incrementally; the dynamic tests involve measurement of a few elements of the frequency response function (FRF) matrix. These measurements are taken to be contaminated by additive Gaussian noise. An artificial independent variable τ, that simultaneously parameterizes the point of application of the moving load, the magnitude of the incrementally varied static load and the driving frequency in the FRFs, is introduced. The state vector is taken to consist of system parameters to be identified. The fact that these parameters are independent of the variable τ is taken to constitute the set of 'process' equations. The measurement equations are derived based on the mechanics of the problem and, quantities, such as displacements and/or strains, are taken to be measured. A recursive algorithm that employs a linearization strategy based on Neumann's expansion of structural static and dynamic stiffness matrices, and, which provides posterior estimates of the mean and covariance of the unknown system parameters, is developed. The satisfactory performance of the proposed approach is illustrated by considering the problem of the identification of the dynamic properties of an inhomogeneous beam and the axial rigidities of members of a truss structure.

Keywords: System identification; Dynamic state estimation; Kalman filter

1. Introduction

The problem of structural system identification lies at the heart of condition assessment of existing structures and in developing structural health monitoring strategies. This class of problems constitutes inverse problems, in which properties of the structure need to be estimated based on noisy data for then applied forces and a limited set of response measurements. These problems are closely associated with problems of finite element (FE) model updating [9,23] and structural damage detection using response data [5]. These problems have received wide attention in the broader context of engineering dynamical systems [8,26,20]. One of the important mathematical tools that form the basis of the development of structural system identification methods is the Kalman filter [19,18,3,13]. The Kalman filter and its variants have been widely used in the development of structural system identification strategies for both linear and nonlinear dynamical systems [35,15,17,10,32]. The Kalman filter provides the exact solution to the problem of state estimation when process and measurement equations are linear and noises are additive and Gaussian. When these conditions are not met, one can develop suboptimal strategies based on linearization or transformation methods [3,27], or, alternatively, employ Monte Carlo simulation strategies to solve the problem [7]. The application of the latter class of approaches to structural engineering problems has been recently attempted by a few authors. Thus, Ching et al. [4], have applied a stochastic simulation based filtering technique, namely, the sequential importance sampling based method as developed by Doucet et al. [6], and an extended Kalman filter

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(EKF) for identifying parameters of three different classes of dynamical systems. Manohar and Roy [21] have applied three simulation-based filtering strategies to the problem of system parameter identification in two typical nonlinear oscillators, namely, the Duffing Coulomb oscillators. The filters that these authors have investigated included: the density based Monte Carlo filter as developed by Tanizaki [33], the Bayesian bootstrap algorithm due to Gordon et al. [12] and the sequential importance sampling based method as developed by Doucet et al. [6]. The application of the Rao–Blackwell theorem in conjunction with a time domain substructuring scheme to identify localized nonlinearities has been investigated by Sajeeb et al. [28]. Namdeo and Manohar [22] have developed a bank of self-learning particle filters for the identification of parameters of nonlinear systems. The recent work by Ghosh et al. [11] outlines a particle filtering procedure to deal with nonlinear measurement models and (or) additive /multiplicative non-Gaussian noises.

Problems of system identification when structural response to only static data is available have been considered by several authors. We cite here a few representative studies. Hoshiya and Sutoh [16] consider problems of system identification in the context of geotechnical engineering problems and combine the finite element method with a Kalman filter-weighted local iteration procedure. Banan etal [1,2] consider the problem of estimating elastic constants of a structural model based on measured displacements under known static loads. These authors minimize an index of discrepancy between experimental analytical model predictions on nodal forces or displacements at measured sites. Sanayei and Saletnik [29] discuss the relative advantages of using strain measurements over measuring displacements, in the context of system identification using static test data. These authors have developed a method that optimizes a quadratic performance error formed using difference between the analytical and measured strains. In a subsequent paper these authors [30] have discussed the accuracy of the system identification *vis-à-vis* the presence of noise in measurements and in the selection of measurement locations. The performance of three alternative identification schemes based on the measurement of strains and displacements has been investigated by Sanayei et al. [31]. This study is based on experiments conducted on a steel frame and demonstrates the successful updating of parameters of the computational model. Yeo et al. [34] illustrate the use of regularization methods in context of damage detection in structures using static test data. Hjelstad and Shin [14] discuss a parameter grouping scheme that enables the identification of the location of structural damage. Paola and Bilello [25] note that a variation in the bending stiffness of a linear elastic beam can be modeled as a superimposed curvature depending on the variation of the flexural rigidity and the applied bending moment. An integral equation based formulation is subsequently proposed for damage detection in such beams. Nejad et al. [24] employ a nonlinear optimization scheme to detect changes in the elastic properties of structures based on static test data. The objective function here is defined in terms of load vectors of damaged and undamaged structures. These authors also propose schemes for the selection of the measurement and driving points that enhance the performance of the damage detection algorithm.

In a condition assessment study of existing engineering structures, it is typically possible to measure the structural response to both statically or dynamically applied loads. Thus, for instance, in the context of the condition assessment of railway bridges, it is possible to measure the structural response when a wagon formation with a known weight distribution can be made to roll across the bridge in a quasi-static manner: this leads to the measurement of influence lines for response quantities such as the strains and displacements at various points on the bridge. Similarly, by parking a wagon on the bridge at a set of specific positions, and, by varying the payload of the wagon, it is possible to measure the response as a function of incrementally varied static load. By allowing a relatively light vehicle to run on the bridge at various velocities the dynamic response of the bridge could be measured. Alternatively, the frequency response functions (FRFs) of the bridge structure can be measured by using modal shakers or an automated sledge hammer. The collection of vibration signatures when operating trains pass the bridge provides data on dynamics of the bridge–train interacting system. The free vibration decay that follows the exit of the trains provides useful data for the estimation of the modal characteristics of the bridge. These data would invariably be spatially incomplete and corrupted by measurement noise. One of the challenges that have to be faced in the problem of the condition assessment of existing bridges, in this context, lies in the ability to handle and assimilate a large amount of noisy measurements in the problem of the identification of system parameters. Specifically, it needs to be appreciated that a part of the data originates from static structural behavior, in which case, the governing equilibrium equations are algebraic in nature; and, conversely, the data from vibration behavior are associated with a set of differential equations in time. The motivation for the present study lies in these considerations and we propose a strategy to assimilate data from diverse testing procedures in a unified manner within the framework of dynamic state estimation procedures. To achieve this, we introduce an artificial independent variable related to the problem on hand and 'sequence' the estimation procedure into a recursive format. The process equations consists of statements that the structural parameters are invariant with respect to the independent variable and the measurement equations are formulated based on the governing equations of equilibrium. A Neumann's expansion of the structural dynamic/static stiffness matrices is further carried out to linearize the measurement equations with respect to the structural parameters of interest. Furthermore, the benefits of adopting a global iteration strategy are also demonstrated. Illustrative examples on the identification of the properties of an inhomogeneous beam (involving 32 system parameters) and the identification of the axial stiffness of elements in a truss structure (involving 25 system parameters) using noisy synthetic data are presented and the methods are shown to perform satisfactorily.