Vacuum 121 (2015) 56–63

Contents lists available at ScienceDirect

Vacuum

journal homepage: www.elsevier.com/locate/vacuum

Torque data on non-isothermal rarefied Cylindrical Couette flow

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ARTICLE INFO

Article history:
Received 25 February 2015
Received in revised form
22 July 2015
Accepted 25 July 2015
Available online 29 July 2015

Keywords:
DSMC
Rarefied cylindrical Couette flow
Isothermal and thermal flows
Gas-surface heat exchange

ABSTRACT

The torque exerted on the walls of a micro cylindrical Couette gas flow is studied using Direct Simulation Monte Carlo (DSMC) method and analytical solutions. An analytical solution for temperature filed under specified wall heat flux condition is proposed using a previously introduced power-law (PL) wall-scaling approach. The results of cylindrical Couette flow under specified wall heat flux condition are compared with those of isothermal flow. The effects of rarefaction, compressibility, tangential momentum accommodation coefficient (TMAC) and gas-surface heat exchange on the torque values are investigated for slip to free molecular flow regimes. The results indicate that the torque coefficient has a maximum value at the transitional flow regime for TMAC values less than unity. Lower TMAC values and higher Knudsen numbers lead to reduction and deviation of torque magnitudes from those of continuum limit. Thermal simulations of cylindrical Couette flow show that heating and cooling have significant effects on the torque values. Velocity profiles and normalized torque values demonstrate mixed increasing/decreasing behavior under heating/cooling processes for rarefied gas flow.

1. Introduction

A true understanding of the gas dynamics and thermal physics of rarefied gas flows is crucial in design, fabrication and operation of micro/nano-electro-mechanical- systems (MEMS/NEMS) [1]. For gaseous flows through MEMS/NEMS, the mean free path is not infinitesimally small with respect to the flow characteristic length. This situation is referred as rarefied condition. Analysis of rarefied gas flows must be performed using accurate approaches based on the Boltzmann equation such as Direct Simulation Monte Carlo method. DSMC is a useful method for analyzing of non-equilibrium rarefied gas flows in which the gas is modeled as a collection of moving particles which interact through collisions [2].

Rarefied gas cylindrical Couette flow has been widely considered analytically and numerically [3–22]. Einzel et al. [3] presented the velocity inversion in rarefied gas cylindrical Couette flow for the first time. Velocity inversion is a phenomenon in which the velocity increases approaching to the outer cylinder meanwhile inner cylinder is rotating and outer one is stationary. It was shown that the velocity inversion occurs at small TMAC values and there was a critical accommodation coefficient [4–18]. Rarefied cylindrical Couette flow was analyzed based on the slip boundary conditions in several works. Effective mean free path models were developed for rarefied cylindrical Couette flow using a power-law-based distribution of mean free path [19,20]. Jung [21] and Dongari et al. [22] considered the effect of various radii ratios on velocity profiles and torque exerted on the rotating inner cylinder. They compared the results of DSMC, power-law and classical slip models. Sharipov and coworkers [23–30] investigated different features of cylindrical Couette flow in rarefied gas. They investigated the effects of rotation on the flow field, viscous stress tensor and heat flux between the cylinders [23–25,27] using BGK model of the Boltzmann equation. They also considered rarefied gas flow between two coaxial cylinders caused by the evaporation and condensation on their surfaces [26]. The equilibrium was perturbed by the small density difference of evaporated particles on the inner cylinder. In another work [28], they analyzed rarefied cylindrical gas flow within the framework of continuum mechanics by considering slip and jump boundary conditions. In the case of significant outer-to-inner radius ratios, they indicated that the surface properties of the outer cylinder weakly affect the heat transfer [29]. In a recent study [30], they investigated the sound waves propagating through a rarefied gas confined between two coaxial cylinders and induced by an unsteady heating of the inner boundary. Pantazis and Valougeorgis [31] and Chalabi et al. [32] investigated numerically and experimentally the heat transfer between coaxial cylinders under temperature difference in rarefied gas. They considered the effects temperature difference at different gas rarefaction and ratio

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http://dx.doi.org/10.1016/j.vacuum.2015.07.017
0042-207X/

of the two cylindrical radii and detected the behavior of the radial heat flow, density, temperature and pressure distributions. It was verified that the type of molecular interaction is important in the presence of strongly nonlinear heat transfer. In an experimental attempt [32], they measured heat conduction through rarefied air at rest contained between two coaxial cylinders under temperature difference of 50–125 K. The heat transferred from the wire (inner cylinder) to the gas was measured for different gas pressure values and consequently different flow rarefactions. The experimental results were compared with the numerical results.

In current study, we investigate the trends of two non-dimensional torque parameters affected from flow rarefaction, tangential momentum accommodation coefficient and gas-surface heat exchange using DSMC method. We also proposed an analytical solution for temperature filed under specified wall heat flux condition using PL model. The topic of rarefied cylindrical Couette flow under specified wall heat flux condition is first presented in this work which enables thermal treatment of such flows for rarefied gas applications such as heat pipes, laser-induced vaporization, material deposition, etc [26]. This topic completes the sequence of previous works in simulation of rarefied gas flows through coaxial cylinders. The behavior of velocity, temperature and density profiles of rarefied gas under heating/cooling conditions is also presented.

2. Computational setup

An extended version of Bird's DSMC1 code developed by Akhlaghi and coworkers [33–37] is employed. In our extended solver, the specified wall heat flux is implemented via Iterative (IT) Technique [33]. In the IT technique, local wall temperature is corrected during the simulation according to the comparison of obtained wall heat flux (q_w) with the desired or expected wall heat flux (q_{des}) as follows:

$$T_w^{new} = T_w^{old}\left(1 + RF\frac{q_w - q_{des}}{|q_{des}| + \varepsilon_0}\right) \tag{1a}$$

RF is a relaxation factor to control the convergence and ε_0 is a positive non-zero energy flux which should be taken negligible in comparison with the incident energy fluxes. It is defined for preventing zero denominator condition in adiabatic wall case. The value of *RF* is set to 0.03. Sampling of results starts as soon as the flow reaches steady state conditions. The steady state condition is achieved when the total number of simulation particles, intermolecular collision rate and macroscopic properties no longer varies appreciably, i.e. less than 1% [37]. Then, flow sampling continues in sufficiently large time step intervals. Generally, statistical noises are diminished using sufficiently large numbers of simulated particles and wide time intervals of sampling. The hard sphere (HS) and generalized hard sphere (GHS) models are applied as collision model for isothermal and thermal simulations of cylindrical Couette flow, respectively. The optimum computational setup for DSMC calculations is employed based on [38,39]. Argon is employed as working fluid for all simulations.

3. Cylindrical Couette flow

Rarefied gas confined between two coaxial rotating cylinders could be considered in four different cases as shown in Fig. 1. Mach number (M) is defined according to the linear velocity of wall and sound velocity in continuum gas, $\sqrt{\gamma k T_w/m}$, in which m is the molecular mass, γ is the specific heat, k is the Boltzmann constant and T_w is the wall temperature. Actually Mach number is employed as a normalized value for wall linear velocity and it is defined for each wall. ω is the wall angular velocity, σ is the value of TMAC, R is the radius of cylinder and q_w is the wall heat flux.

The values of inner radius (R_1) and ratio of R_2/R_1 are set to be 1.0 μm and 2.0, respectively. Mach number for all moving walls is considered to be equal to 0.1. Different values of TMAC are considered, i.e. 0.1, 0.5 and 1. Flow rarefaction is defined as the ratio of mean free path to gap distance between two cylinders:

$$Kn = \frac{\lambda}{R_2 - R_1} \tag{1b}$$

The mean free path MFP (λ) is calculated based on the average density. Assuming fully developed flow, the torque exerted on the walls (τ) is

$$\tau_1 = 2\pi R_1^2 \delta|_{r=R_1}$$
$$\tau_2 = 2\pi R_2^2 \delta|_{r=R_2} \tag{2}$$

where δ is the wall shear stress. Wall shear stress and consequently, torque exerted on the wall are functions of flow rarefaction.

3.1. Isothermal Cylindrical Couette flow

Dongari et al. [18,20,22] proposed analytical solution for isothermal slip and transitional rarefied gas flows between coaxial cylinders based on power-law (PL) wall-scaling approach as follows

$$u_\theta(r) = r\left[\omega_1 + \frac{\alpha_1 \lambda C}{R_1^3} + G(r)C\right] \tag{3}$$

where

$$G(r) = \int_{R_1}^{r} \frac{dr}{r^3\beta} \tag{4}$$

$$C = \frac{\omega_2 - \omega_1}{H + \frac{\alpha_1\lambda}{R_1^3} + \frac{\alpha_2\lambda}{R_2^3}} \tag{5}$$

$$H = \int_{R_1}^{R_2} \frac{dr}{r^3\beta} \tag{6}$$

r and θ correspond to the radial and tangential coordinates. β is the geometry-dependent normalized MFP which considers nonlinear flow physics in the near-wall regions, i.e., $\lambda_{eff}=\beta\lambda$. It is a function of Knudsen number and radial position in the gap, i.e., $\beta(Kn,r)$. The details of β formulation has been presented in Refs. [18,20,22]. Based on PL model, wall shear stress is defined as

$$\delta = \underbrace{\mu\beta}_{\mu_{eff}}\left(\frac{du_\theta}{dr} - \frac{u_\theta}{r}\right) \tag{7}$$

where μ is the fluid viscosity. In molecular approaches, e.g. DSMC method, the magnitude of wall shear stress is calculated by sampling over incident and reflected momentum of interacting particles with wall:

$$\delta_{DSMC} = \frac{\sum m\left(V_t^i - V_t^r\right)}{t_s A} \tag{8}$$

where t_s, A and V are sampling time, area and tangential velocity