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Unsteady rarefied gas flow through a slit

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ABSTRACT

A transient gas flow through a rectangular slit is simulated on the basis of unsteady S-model kinetic equation. The flow into vacuum and at two pressure ratios between the reservoirs are considered for a wide range of the rarefaction parameter lying from the free molecular to the near continuum regime. The time of steady flow establishment is obtained and it is found that this time depends on the pressure ratio between the reservoirs and on the gas rarefaction. The time evolution of the macroscopic flow parameters is analyzed in detail. The present results are compared with the results on the orifice flow previously obtained by other authors.

1. Introduction

The gas flow through a slit into vacuum or at arbitrary pressure ratio is a fundamental problem of the rarefied gas dynamics. The flow through a slit was proposed recently [1] as a benchmark to test different approaches for the rarefied gas flow simulations, like numerical solution of the Boltzmann equation or its kinetic models and the DSMC method. For many practical applications such as molecular beam, nozzle flows, surface coating inside vacuum chambers, information about the evolution of the flow parameters between two tanks connected by a slit is indispensable.

The detailed review of a rarefied gas flow through a slit was recently given in Ref. [2]. However, despite more than forty-year history of slit flows studies only the steady state formulation was considered. Meanwhile, in a practice an essential interest presents the transient gas flow properties. Especially an information about the rapidity of the steady state flow establishment may be useful in vacuum metrology [3,4], for the gauge calibration and control of their accuracy [5], for the flow meters development and for the vacuum mass metrology [6]. Detailed information about the time dependent gas flows characteristics is also required for vacuum pumps testing or quantification of leak rates [6].

Recently several papers are published, where the transient flows through an orifice [7], short [8] and long tubes [9] are studied.

The aim of the present paper is an analysis of the transient gas flow through a slit. The time dependent flow characteristics are obtained by solving numerically the unsteady S-model kinetic equation using the Discrete Velocity Method (DVM).

2. The problem statement

Let us consider two reservoirs maintained at different pressures (Fig. 1) and separated by an infinitesimally thin wall with a slit. The slit height is equal to H in the y direction and it is supposed to be infinite in the z-direction, the z axis is oriented perpendicularly to the plane of Fig. 1. The pressure and temperature in the left reservoir are equal to p_0 and T_0, respectively, while the right reservoir is maintained at the pressure p_1 ($p_1 < p_0$) and at the same temperature $T_1 = T_0$.

It was shown in Ref. [10] that the gas–surface interaction does not have any influence on a slit flow, therefore this flow can be completely characterized by only two parameters:

- rarefaction parameter δ defined as follows

$$\delta = \frac{p_0 H}{\mu_0 v_0}, \quad v_0 = \sqrt{2\frac{k}{m}T_0}, \tag{1}$$

here m is the mass of a molecule, k is the Boltzmann constant, μ_0 is the viscosity coefficient calculated for the reference temperature T_0, v_0 is the most probable molecular velocity.

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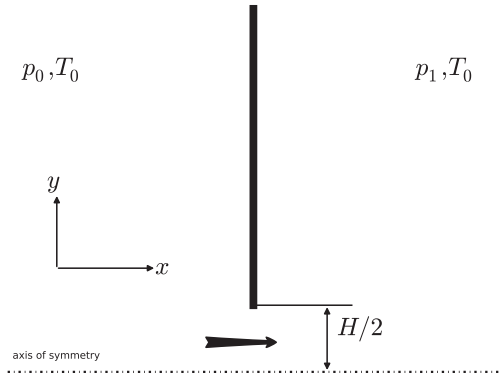

Fig. 1. Scheme of the flow and coordinates.

- pressure ratio p_1/p_0.

The S-model kinetic equation [11] is used for the numerical simulation of the transient flow behavior

$$\frac{\partial f'}{\partial t'} + \mathbf{v}\cdot\frac{\partial f'}{\partial \mathbf{r}'} = \frac{(f^{S'} - f')}{\tau'}, \tag{2}$$

here $f'(t',\mathbf{r}',\mathbf{v}')$ is the unknown molecular velocity distribution function, t' is the time, $\mathbf{v}$ is a molecular velocity, $\mathbf{r}'$ is the position vector, τ' is the relaxation time, $f^{S'}$ is the equilibrium distribution function defined in Ref. [11] as

$$f^{S'}(n',T',\mathbf{u}') = f^M\left[1 + \frac{2m}{15n'(t',\mathbf{r}')(kT'(t',\mathbf{r}'))^2}\mathbf{V}\cdot\mathbf{q}'(t',\mathbf{r}') \times \left(\frac{mV^2}{2kT'(t',\mathbf{r}')} - \frac{5}{2}\right)\right], \tag{3}$$

where f^M is the local Maxwellian distribution function, $\mathbf{u}'$ is the bulk velocity, $\mathbf{V} = \mathbf{v} - \mathbf{u}'$ is the peculiar velocity, $\mathbf{q}'$ is the heat flux vector, T' is the gas temperature, n' is the gas number density.

The main characteristic of the slit flow is the mass flow rate $\dot{M}$ per unit length in the z-direction which may be defined as

$$\dot{M} = 2m\int_0^{H/2} n'u_x' \mathrm{d}y', \tag{4}$$

where u_x' is the x component of the bulk velocity $\mathbf{u}'$. In the free molecular regime ($\delta = 0$), the mass flow rate through the slit into vacuum ($p_1/p_{0=0}$) per unit length in the z-direction was calculated analytically in Ref. [12]

$$\dot{M}_{FM} = \frac{p_0 H}{\sqrt{\pi}v_0}. \tag{5}$$

In the following the numerical results for the transient mass flow rate $\dot{M}$ at several pressure ratios and for different rarefaction parameters will be given in the normalized form

$$W = \frac{\dot{M}}{\dot{M}_{FM}}. \tag{6}$$

As it has been mentioned previously, the gas–surface interaction does not have any influence on the flow when the diffuse reflection of the molecules from the surface is assumed.

3. The method of solution

Following Refs. [13,2] for the numerical solution of the S-model kinetic eq. (2) the dimensionless quantities are introduced:

$$\mathbf{r} = \frac{\mathbf{r}'}{H}, \quad \mathbf{c} = \frac{\mathbf{v}}{v_0}, \quad T = \frac{T'}{T_0}, \quad n = \frac{n'}{n_0}, \quad \mathbf{u} = \frac{\mathbf{u}'}{v_0}, \tag{7}$$

$$t = \frac{t'}{t_0}, \quad p = \frac{p'}{p_0}, \quad \mathbf{q} = \frac{\mathbf{q}'}{p_0 v_0}, \quad \mu = \frac{\mu'}{\mu_0}, \quad f = f\frac{{v_0}^3}{n_0}. \tag{8}$$

Here the state equation in the form $p_0 = n_0 k T_0$ is implemented, the relaxation time τ is calculated from relation $\tau = \mu/p$. The characteristic time of the flow t_0 is introduced in (8) and is equal to

$$t_0 = \frac{H}{v_0}. \tag{9}$$

For the dependence of the viscosity coefficient from the temperature the hard sphere model is used, therefore $\mu = \sqrt{T}$.

Using dimensionless variables (7), (8) and taking into account the two dimensional character of the problem the model kinetic eq. (2) becomes

$$\frac{\partial f}{\partial t} + c_x\frac{\partial f}{\partial x} + c_y\frac{\partial f}{\partial y} = \delta n\sqrt{T}\left(f^S - f\right). \tag{10}$$

We suppose that the flow characteristics do not vary in the z-direction when the macroscopic parameters do not depend on the z coordinate. Therefore, to decrease the number of variables in the velocity space we can use the classical projection procedure introducing the reduced distribution functions:

$$\Phi(t,x,y,c_x,c_y) = \int_{-\infty}^{\infty} f\mathrm{d}c_z, \quad \Psi(t,x,y,c_x,c_y) = \int_{-\infty}^{\infty} f{c_z}^2\mathrm{d}c_z, \tag{11}$$

so kinetic eq. (10) boils down to the set of two kinetic equations with respect to the functions Φ, Ψ:

$$\begin{aligned}\frac{\partial \Phi}{\partial t} + c_x\frac{\partial \Phi}{\partial x} + c_y\frac{\partial \Phi}{\partial y} &= \delta n\sqrt{T}\left(\Phi^S - \Phi\right),\\ \frac{\partial \Psi}{\partial t} + c_x\frac{\partial \Psi}{\partial x} + c_y\frac{\partial \Psi}{\partial y} &= \delta n\sqrt{T}\left(\Psi^S - \Psi\right).\end{aligned} \tag{12}$$

On the boundaries of the computational domain, far away from the slit, the distribution function for the incoming molecules is taken to be equal to the absolute Maxwellian distribution with the parameters determined by the pressure and temperature of each reservoir and with the bulk velocity equal to zero. On the solid walls, which separate two reservoirs, the impermeability conditions are used to determine the number density in the left and right reservoirs, see Refs. [13,2] for details.

At $t = 0$ the macroscopic parameters in each reservoir are equal to p_0, T_0 and to p_1, T_0, respectively and the flow velocity is supposed to be zero. At an initial time moment the slit is opening and the gas starts to flow through the slit from the left reservoir to the right one.

Due to the flow symmetry the calculations are carried out only for upper part of the physical space, $y \geq 0$, see Fig. 1. The influence of the computational domain dimensions on the flow parameters was studied in Refs. [13,2]. Using these results the dimensions of the left and right reservoirs are chosen to be equal to $100H \times 100H$, however in the case of $p_1/p_0 = 0$ (gas expansion into vacuum) the smaller dimensions of the right reservoir are used, equal to $10H \times 10H$. The detailed study of the grid size influence has been carried out in Refs. [13,2], therefore these previous results were