Mechanism and Machine Theory 101 (2016) 60–74

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Dynamic tooth root strains and experimental correlations in spur gear pairs

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ARTICLE INFO

Article history:
Received 9 December 2015
Accepted 13 March 2016
Available online 31 March 2016

Keywords:
Gear pair
Static strain
Contact condition
Dynamic strain
Vibration

ABSTRACT

This work investigates the static and dynamic tooth root strains in spur gear pairs using a finite element/contact mechanics approach. Extensive comparisons with experiments, including those from the literature and new ones, confirm that the finite element/contact mechanics formulation accurately predicts the tooth root strains. The model is then used to investigate the features of the tooth root strain curves as the gears rotate kinematically and the tooth contact conditions change. Tooth profile modifications are shown to strongly affect the shape of the strain curve. The effects of strain gage location on the shape of the static strain curves are investigated. At non-resonant speeds the dynamic tooth root strain curves have similar shapes as the static strain curves. At resonant speeds, however, the dynamic tooth root strain curves are drastically different because large amplitude vibration causes tooth contact loss. There are three types of contact loss nonlinearities: incomplete tooth contact, total contact loss, and tooth skipping, and each of these has a unique strain curve. Results show that different operating speeds with the same dynamic transmission error can have much different dynamic tooth strain.

1. Introduction

Gear teeth can fail as a result of dynamic loads causing large tooth root stresses and strains. Understanding and predicting these stresses and strains remain a challenging task for engineers trying to avoid structural failure.

Gear dynamics has been studied for decades using lumped-parameter models, that is, when the gears are modeled as discrete masses and moments of inertia connected by stiffness elements. Özgüven and Houser [1] reviewed mathematical models for gear dynamics prior to 1988. Gear dynamics studies prior to 1996 can be found in the comprehensive bibliography of Ref. [2]. Wang et al. [3] reviewed gear dynamics and vibration with a focus on nonlinear behavior. In an early work, Gregory et al. [4] investigated the vibration of spur gear pairs analytically and experimentally. Blankenship and Singh [5,6] developed a three-dimensional gear mesh model to study the vibration of helical gears. Velex and Maatar [2] derived a gear mesh interface model by discretizing the instantaneous lines of tooth contact. Their model includes the effects of tooth surface modifications and errors. Eritenel and Parker [7,8] investigated nonlinear vibration in helical gears using a three-dimensional

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http://dx.doi.org/10.1016/j.mechmachtheory.2016.03.010

lumped-parameter model. Their tooth mesh interface is modeled as a discretized network of stiffnesses that captures total and partial contact loss (where portions of the nominal lines of contact lose contact). Palermo et al. [9] derived a three-dimensional mesh interface model intended for multibody dynamics simulations. Eritenel and Parker [10] derived closed-form solutions for the nonlinear vibration of gear pairs with total and partial contact loss using a perturbation method. Lumped-parameter models, like those cited above, assume the gears are rigid bodies. Tooth root strains are not directly calculated using these models. Dynamic tooth root stresses are calculated in Refs. [11–13] using analytical expressions for tooth root stresses that depend on dynamic tooth forces from the lumped-parameter model.

A finite element/contact mechanics formulation for the elastic contact between gear teeth has been developed by Vijayakar and co-workers [14–16]. This formulation can also be used to model the elastic contact of the roller/race interface in rolling element bearings [17]. Parker et al. [18] determined the dynamic response for two-dimensional planetary gears using this formulation. Parker et al. [19] showed that the finite element/contact mechanics formulation accurately predicts the vibration in gear pairs compared with experiments, including tooth contact loss and the nonlinear jump-up and jump-down frequencies near resonance. Tamminana et al. [20] used the finite element/contact mechanics model to study the relationship between the dynamic factor and dynamic transmission error for spur gear pairs. Cooley et al. [21] developed a frequency domain finite element method for the dynamic analysis of three-dimensional gear systems. These works focus on rigid body gear deflections, especially dynamic transmission error and dynamic tooth forces. In contrast, tooth root strains are a focus of the current work.

Tooth root stresses have been studied experimentally using photo-elastic methods for spur gear pairs in Refs. [22,23] and planetary gears in Ref. [24]. Photo-elastic methods show stress distributions and locations of the maximum stress. Strain gages are used to measure tooth root strains for gear pairs in Refs. [11,25–29] and planetary gears in Refs. [30–34].

Regarding calculation of tooth root stresses and strains, numerous models have been used. Ichimaru and Hirano [11] calculated the static and dynamic tooth root strains in spur gears using an analytical model. Chabert et al. [35] calculated spur gear tooth root stresses using a two-dimensional finite element model of a single tooth with a concentrated load applied to its surface. Oda et al. [36] investigated the static tooth root stresses in thin-rimmed spur gears using a finite element model of only a few teeth with a concentrated load at the tooth surface. Oda et al. [37] used a boundary element method to calculate tooth stresses in thin-rimmed spur gears. Lee et al. [12] investigated the effects of profile modifications on gear tooth root stresses by substituting the dynamic tooth forces calculated from an analytical model into an analytical formula (the Heywood formula) for tooth root stresses. Lin et al. [13] discussed the design of profile modifications to reduce vibration, tooth loads, and root stresses. Bibel et al. [38] calculated the root stresses for thin-rimmed spur gears using a finite element model. They investigated the sensitivity of the stress calculations to the boundary conditions of the model. Baud and Velex [26] analytically investigated static and dynamic tooth root stresses in gear pairs. Their model agreed with measurements. Wang and Howard [39] calculated the static stresses for high contact ratio spur gears using a conventional finite element method. Kawalec et al. [40] compared the tooth root stresses from a finite element model to ISO and AGMA standards. Thirumurugan and Murthuveerappan [41] investigated the effects of gear parameters on the static tooth stresses in spur gears using a finite element model. Palmer and Fish [42] compared analytical and finite element models for tooth root stresses. The effect of tip relief and involute contact ratio on static tooth root stresses is investigated. Kahraman and Vijayakar [43] calculated the tooth root stresses in planetary gears using a finite element/contact mechanics model. Singh et al. [30] correlated static tooth root and hoop strains calculated from the finite element/contact mechanics formulation with experiments for the ring gears of planetary gears. The finite element/contact mechanics model is used to investigate the effect of ring gear rim thickness on static tooth root and hoop strains in Ref. [31]. None of these studies have focused on the shape of the static and dynamic strain curves, which is a focus of this work.

In this work, we investigate the static and dynamic gear tooth root strains using a finite element/contact mechanics model. Two different unity-ratio gear pairs are studied: a 28-tooth gear pair with asymmetric teeth and a 50-tooth gear pair. The finite element calculations of static and dynamic strains are correlated to experiments for both gear pairs. The correlated finite element models are used to investigate the features of the static and dynamic strain curves. The shapes of the static strain curves are explained using the changing contact conditions on the gear teeth. The effects of tooth profile modifications and strain gage location on the static strain curves are investigated. The shapes of the dynamic strain curves with three types of contact loss nonlinearities are explained using dynamic contact conditions and dynamic tooth loads. The effects of incomplete tooth contact, defined as one of the two tooth pairs normally in contact losing contact, and total contact loss due to large amplitude vibrations are explained. Even without any form of contact loss, different speeds that have the same amplitude of dynamic transmission error (DTE) can have much different amplitudes of dynamic tooth strain.

2. Finite element/contact mechanics formulation

We use a finite element/contact mechanics formulation to calculate the static and dynamic tooth root strains in spur gear pairs. The formulation is described by Vijayakar and co-workers in Refs. [14–16] and summarized for gear dynamics applications in Refs. [19,21]. A concise overview of the contact mechanics model can be found in Ref. [18]. We briefly describe the formulation below.

The finite element/contact mechanics model is built on an underlying multibody dynamics formulation that captures the nominal gear rotation at an operating speed. Each gear body has a finite element mesh for elastic deformations and six rigid body