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The mathematical model and mechanical properties of variable center distance gears based on screw theory

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ABSTRACT

Currently, researches on planar gears (including cylindrical gear, noncircular gear) are primarily concerned with those of fixed center distance, that is, the center distance between two meshing gears remains unchanged during the driving process, while those of variable center distance are rarely touched upon. Although not widely applied as fixed center distance gear is, variable center distance gear can be found in noncircular gear shaping and high-torque hydraulic motor, exhibiting unique mechanical properties.

This paper focuses on a systematic revelation of the driving principle and mechanical properties of this special variable center distance gear type. On the basis of screw theory, this paper first establishes its mathematical model, including kinematic relations, instantaneous screw axis, "non-conjugated" pitch curves, generator, envelope surfaces and generated surfaces. And then, the mechanical properties, forms as well as advantages of applicable variable center distance gear train are all discussed.

1. Introduction

With compact structure, accurate non-uniform transmission and unique dynamic performance, various applications of non-circular gear have been developed: Emura and Arakawa [1] utilized a non-circular gear for steering mechanism analysis; Dooner [2] used non-circular gears to reduce speed and torque fluctuations in rotating shafts; Litvin et al. [3] investigated the gear drives with non-circular gears applied for speed variation and generation of functions; Ottaviano et al. [4] analyzed the non-circular gears and cam-follower systems as function generators; H. Terada et al. [5] developed a knee motion assist mechanism for wearable robot with a non-circular gear and grooved cams; Liu et al. [6] designed noncircular gears utilized in polishing mechanisms of optical fiber end-face polishing machines to perform a figure-8 pattern. D. Mundo [7] developed a planetary gear train with non-circular gears in the design of a power drive mechanism for high performance bicycles. K.-H. Modler et al. [8] proposed a general method for the synthesis of geared linkages with non-circular gears.

Other researches, meanwhile, have extensively focused on the generation of noncircular gear: Tong, S.H., and Yang, C. H. D. [9] investigated the generation of identical noncircular pitch curves; Litvin et al. [10] discussed the generation of planar and helical elliptical gears by application of rack-cutter, hob, and shaper; Figliolini, G. and Angeles, J. synthesized the elliptical gears generated by shaper cutters [11] and the base curves for n-lobed elliptical gears [12]; Bair, B. W. et al. established the mathematical model of elliptical gears manufactured respectively by a commonly known shaper cutter [13] and a circular-arc shaper cutters [14]; Xia, L. et al. [15] proposed a linkage model of hobbing non-circular helical gears with axial shift of hob; Jiangang, L. et all [16] introduced a numerical computing method for noncircular gear tooth profiles generated by shaper cutters.

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Based on these researches, noncircular gear is commonly defined as gear with fixed center distance, in which the center distance of gear is set as a constant and remains unchanged during the driving process. In fact, in the process of generating noncircular gear with shaper cutter, as discussed by Litvin et al. [3], Figliolini [11] and Dooner [17], a circular cutter is in mesh with a planar non-circular gear element and their center distance varies. The meshing process can thus be taken as a case of variable center distance gear (VCDG) drive; Besides, in a particular type of noncircular planetary gear hydraulic motor [18,19], the center distance of the planetary gear changes during the driving process, making it a typical VCDG drive. Overall, although researches on this gear type have been found as listed above, the concept of VCDG has not been definitely proposed.

This paper then devotes to a systematic investigation of mathematical model and mechanical properties of this special VCDG type: First, establishes the generating mathematic model: kinematic relations, instantaneous screw axis, "non-conjugated" pitch curves, generator, envelope surfaces and the generated surfaces; then drawn on three cases of special gearing and triple methods of substitution, discusses the mechanical properties; finally, combing the mathematic model and mechanical properties, develops three gear train types with applicable advantages.

2. Mathematic model for VCDGs

2.1. Kinematic relations

Fig. 1 shows the kinematic relationship of shaping a non-circular gear in a 3-axis linkage shaping machine. The rotating axis of shaped gear is fixed on the base, while that of shaper cutter is movable along the line of centers. During the shaping process, shaped gear rotates on its axis with an angular velocity ω_1, while the shaper cutter simultaneously rotates on its axis with an angular velocity ω_2 and translates along the line of centers. This process can also be recognized as special gear transmission since the shaper cutter can be taken as a cylindrical gear. The peculiarity of this gearing lies in a varying center distance during the driving process, distinguishing it from other cylindrical gear driving.

In generalization of the gear shaping case, the concept of variable center distance gears (VCDGs) is proposed in the paper. Suppose the rotating angle of drive gear is φ_1, then the gear ratio function is $g(\varphi_1)=\omega_2/\omega_1$ and the translational velocity of movable gear (defined as driven gear in this paper) is defined as $v(\varphi_1)$. Noticeably, drive gear φ_1 is supposed in the clockwise direction, the positive direction of translation is defined as far away from the fixed center gear, and the signal of gear ratio is set as positive when the rotating directions of drive and driven gear are the same (i.e. internal gear pair).

The positional relations, including the angle of driven gear $\varphi_2(\varphi_1)$ and center distance function $E_2(\varphi_1)$, can then be calculated

$$\begin{cases} \varphi_2(\varphi_1) = \displaystyle\int_0^{\varphi_1} g(\varphi)\mathrm{d}\varphi \\ E(\varphi_1) = \displaystyle\int_0^{\varphi_1} v(\varphi)\mathrm{d}\varphi + E_0 \end{cases} \tag{1}$$

where, the initial value for $E(\varphi_1)$ is supposed as E_0, the initial value for $\varphi_2(\varphi_1)$ is set as zero and can thus be omitted.

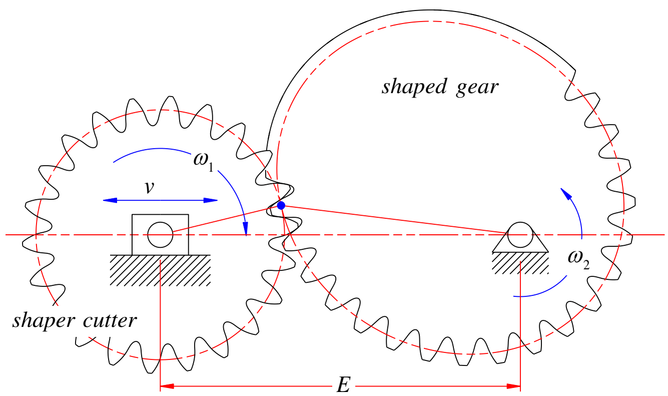

Fig. 1. Case of VCDG: shaping non-circular gear.