Mechanism and Machine Theory 101 (2016) 140–157

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Design and kinematics evaluation of a gear pair with asymmetric parabolic teeth

Ying-Jen Chen [a], Hsueh-Cheng Yang [a,*], P. Frank Pai [b]

[a] Graduate Institute of Vehicle Engineering, National Changhua University of Education, Changhua 500, Taiwan, ROC
[b] Department of Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, USA

A R T I C L E I N F O

Article history:
Received 8 June 2015
Received in revised form 24 February 2016
Accepted 9 March 2016
Available online 2 April 2016

Keywords:
Asymmetric parabolic tooth profile
Imaginary rack cutter
Tooth contact analysis

A B S T R A C T

This paper presents design of imaginary spur and skew rack cutters having teeth of asymmetric parabolic profiles to generate spur and helical gear pairs, respectively. First, a mathematical model of the imaginary rack cutter having teeth of an asymmetric parabolic profile is derived using geometric relations and coordinate transformation. After the relationship between the coordinate system of the imaginary rack cutter and that of the gear pair is set up, a family of imaginary rack cutter surfaces is obtained using the homogeneous coordinate transformation matrix to transfer the coordinate system of the rack cutter to that of the gear pair. Substituting the equations of meshing into the family of imaginary rack cutter surfaces, a pinion and a gear are generated and their geometries are plotted using an in-house software package. Based on the derived mathematical model of the gear pair with assumed assembly errors, tooth contact analysis is performed to determine the influences of kinematic errors and the asymmetric tooth profile. A helical gear pair made by CNC machining is used to verify the influences of asymmetric parabolic tooth profiles on contact conditions and kinematic performance.

1. Introduction

Popular tooth profiles include involute, cycloid and circular curves, but involute gears are more commonly used in many areas. Gears can be created by gear hobbing, gear shaping and gear grinding. Gear hobbing uses an imaginary rack cutter. Research in the literature on the tooth profiles of rack cutters is mainly on symmetric and asymmetric involute profiles, cycloid profile, and symmetric parabolic profile. For example, Kapelevich [1], Pedersen [2] and Litvin [3,4] investigated the use of asymmetric involute spur gears to increase load-carrying capacity and reduce vibration noise and weight. A parabolic curve was used to crown the work region of a gear in [5]. Zhang and Gauo [6] studied the generation of parabolic teeth and examined the pointwise contact properties of Litvin's parabolic gear pair [3,4]. Zanzi and Pedrero [7] presented the use of gears with surfaces being modified to be parabolic using a grinding disk during the generation of the matching pinion.

The tooth contact analysis (TCA) method proposed by Litvin et al. [8] is used here to analyze the contact condition and kinematic errors of helical parabolic gear pairs under assumed assembly errors. Chen et al. [9] performed simulations on the contact condition of helical circular-arc gear pairs under assumed assembly errors. Yang [10] investigated the contact condition of gear pairs created by rack cutters of asymmetric involute tooth profiles. Tsay [11], Simon [12] and Li [13] used the TCA method to study the contact condition of helical involute gear pairs and found that edge contact may happen during gear meshing. However, the contact condition of asymmetric parabolic gear pairs created by rack cutters and under assembly errors has not been studied in the literature.

* Corresponding author.
E-mail address: scyang57@cc.ncue.edu.tw (H.-C. Yang).

http://dx.doi.org/10.1016/j.mechmachtheory.2016.03.008
0094-114X/

Nomenclature

a_c and a_t design parameters of the rack cutter.
c parabolic coefficient
m_n normal module
r_1 and r_2 the radii of the root fillets
r_3 and r_4 standard pitch radii of pinion and gear, respectively.
t_i and u curvilinear coordinates of rack cutter where subscript $i = 1–9$.
$\hat{t}$ and $\hat{u}$ curvilinear coordinates of the helical pinion.
$\mathbf{M}_{32}$ and $\mathbf{M}_{42}$ co-ordinate transformation matrix from co-ordinate system $x_2y_2z_2$ to $x_3y_3z_3$ and $x_2y_2z_2$ to $x_4y_4z_4$, respectively.
N_4 and N_3 numbers of teeth for the gear and pinion, respectively.
$N_2^{(i,j)}$ vector normal to the imaginary skew rack cutter
$(\overline{o_2o_j})_2$ the distance between o_j and o_2 presented in terms of the coordinate system $x_2y_2z_2$.
$\mathbf{R}_1^{(i,j)}$ the position vector of the proposed rack cutter where The first superscript i indicates the segments $\overline{ab}$, $\overline{bc}$, $\overline{cd}$, $\overline{de}$, $\overline{fg}$, $\overline{gh}$, $\overline{hi}$, $\overline{ij}$, $\overline{ak}$, $\overline{k\ell}$ and $\overline{lf}$, and the second superscript j indicates a pinion ($j = 3$) or a gear ($j = 4$).
$\mathbf{R}_2^{(i,j)}$ the position vector of the skew imaginary rack cutter.
$x_iy_iz_i$ coordinate systems where subscript $i = 1, 2, 3, 4, 5$ and 6, 1 denotes the normal section of the imaginary skew rack cuter. $2, 3, 4$ and 5 are rigidly fixed to the imaginary skew rack cutter, the pinion, the gear and the gear housing, respectively. 6 denotes an auxiliary coordinate system.
V_2^{2j} the relative velocity between the gear (or pinion) and the rack cutter represented in coordinate system $x_2y_2z_2$.
$\boldsymbol{\omega}_2^{2j}$ the relative angular velocity between the gear (or pinion) and the skew rack cutter.
β helix angle.
Σ_3^t and Σ_4^t the rack cutters were used to create pinion and gear profiles, respectively.
θ_1 and θ_2 pressure angles.
ϕ_3 and ϕ_4 the gear and the pinion rotate about its own axis by an angle, respectively.
$\overline{\phi}_3$ and $\overline{\phi}_4$ rotary angles in TCA.
Δc the error of the center distance between points O_3 and O_4.
γ_1 and γ_2 the horizontal and the vertical misalignment angles, respectively.
K.E. kinematic errors.

In this paper, a mathematical model a rack cutter having asymmetric parabolic tooth profile is derived by using geometric relations. Based on the obtained mathematical model an imaginary rack cutter is drawn by using *Mathematica*. Using the imaginary rack cutter as the normal section of a skew rack cutter one can derive the imaginary skew rack cutter having an asymmetric parabolic tooth profile through coordinate transformation. Based on Litvin's theory of gearing [14,15], a family of imaginary skew rack-cutter surfaces and an equation of meshing between the skew rack-cutter and the gear blank are used to create mathematical models of a helical gear pair. Assembly errors may occur in the assembly of the proposed helical gear and pinion having asymmetric parabolic tooth profiles and induce a kinematic error between the gear pair. The contact condition between the helical gear pair having asymmetric parabolic tooth profiles is first evaluated using *SolidWorks*. The kinematic errors of the proposed helical gear pair can be calculated by using the tooth contact analysis (TCA) method. Kinematic errors due to axial misalignment and center distance error are simulated in a mating gear set. A pair of the proposed helical gears having asymmetric parabolic tooth profiles was manufactured by using CNC machining and was used to demonstrate real tooth contact conditions. The helical gears are set up with assembly errors, and actual contact conditions are studied in detail. One advantage of the present method is the ability to provide a rapid and simple geometric model of a gear with asymmetric parabolic profile. Compare to symmetric parabolic tooth, circular-arc tooth and involute tooth, this asymmetric parabolic gear type can reduce the size and the weight of gear and increase its load capacity. The mathematical models and analysis methods proposed for the helical gear mechanism with asymmetric parabolic teeth should be useful for the design and production of helical gears with asymmetric parabolic teeth.

2. The normal section of the imaginary skew rack cutter

In this study, geometric relations and parameters are used to derive the asymmetric parabolic tooth profile, and the mathematical model of the designed asymmetric parabolic tooth profile is confirmed using the *Mathematica* software. In Fig. 1, the segments $\overline{ab}$, $\overline{bc}$, $\overline{cd}$, $\overline{de}$, $\overline{fg}$, $\overline{gh}$, $\overline{hi}$ and $\overline{ij}$ are for creating a gear, and the segments $\overline{ab}$, $\overline{bc}$, $\overline{fg}$, $\overline{gh}$, $\overline{ak}$, $\overline{kl}$ and $\overline{lf}$ are for creating a pinion. Here, the segments $\overline{ab}$, $\overline{bc}$, $\overline{fg}$ and $\overline{gh}$ represent the flank of rack cutter are used to create the work region of a gear pair. Segments $\overline{cd}$ and $\overline{hi}$ on the normal section of the skew rack cutter generate the fillet surface of the gear blank. Similarly, segments $\overline{ak}$ and $\overline{lf}$ on the normal section of the skew rack cutter generate the fillet surface of the pinion blank. Based on Fig. 1, the equations for all segments of the rack cutter having an asymmetric parabolic tooth profile can be presented in terms of $x_1y_1z_1$ the coordinate system as shown next.