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Continuous approximate synthesis of planar function-generators minimising the design error

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ABSTRACT

It has been observed in the literature that as the cardinality of the prescribed discrete input–output data set increases, the corresponding four-bar linkages that minimise the Euclidean norm of the design and structural errors tend to converge to the same linkage. The important implication is that minimising the Euclidean norm, or any p-norm, of the structural error, which leads to a nonlinear least-squares problem requiring iterative solutions, can be accomplished implicitly by minimising that of the design error, which leads to a linear least-squares problem that can be solved directly. Apropos, the goal of this paper is to take the first step towards proving that as the cardinality of the data set tends towards infinity the observation is indeed true. In this paper we will integrate the synthesis equations in the range between minimum and maximum input values, thereby reposing the discrete approximate synthesis problem as a continuous one. Moreover, we will prove that a lower bound of the Euclidean norm, and indeed of any p-norm, of the design error for planar RRRR function-generating linkages exists and is attained with continuous approximate synthesis.

1. Introduction

Design and structural errors are important performance indicators in the assessment and optimisation of function-generating linkages arising by means of approximate synthesis. The *design error* indicates the error residual incurred by a specific linkage in satisfying its synthesis equations. The *structural error*, in turn, is the difference between the prescribed linkage output value and the actual generated output value for a given input value [1]. From a design point of view it may be successfully argued that the structural error is the one that really matters, for it is directly related to the performance of the linkage.

It was shown in Ref. [2] that as the cardinality of the prescribed discrete input–output (I/O) data-set increases, the corresponding linkages that minimise the Euclidean norms of the design and structural errors tend to converge to the same linkage. The important implication of this observation is that the minimisation of the Euclidean norm of the structural error can be accomplished indirectly via the minimisation of the corresponding norm of the design error, provided that a suitably large number of I/O pairs is prescribed. The importance arises from the fact that the minimisation of the Euclidean norm of the design error leads to a linear least-squares problem whose solution can be obtained directly as opposed to iteratively [3,4], while the minimisation of the same norm of the structural error leads to a nonlinear least-squares problem, and hence, calls for an iterative solution [1].

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Several issues have arisen in the design error minimisation of four-bar linkages. First, the condition number of the synthesis matrix may lead to design parameters that poorly approximate the prescribed function [5]. This problem can be addressed through careful selection of the I/O pairs used to generate the synthesis matrix. It has also been suggested to introduce dial zeros whose values are chosen to minimise the condition number of the synthesis matrix [6]. Second, the identified design parameters have a dependence on the I/O set cardinality. As the number of I/O pairs grows, the magnitude of the design error tends to converge to a lower bound. Hence, the I/O set cardinality might be fixed as soon as the magnitude of the design error reaches some pre-defined minimum value [2].

Diverse interesting and useful optimisation strategies have been proposed recently for structural error minimisation in planar four-bar function-generators. For example, in Ref. [7] the authors define the least squares error between the desired and generated functions as the objective function for a sequential quadratic programming (SQP) approach. The proposed method solves a sequence of optimisation subproblems, each of which optimises a quadratic model of the objective function subject to a linearisation of the constraints based on the distribution of a finite set of precision points. Another novel approach which considers the minimisation of the structural error of the link lengths is described in Ref. [8]. The method treats one of the dyads as having fixed distances between joint centres, while the other dyad has links of variable length. The adjustable link lengths are varied using a discrete set of precision points as benchmarks. A completely different approach is used in Ref. [9] to develop a probabilistic, time-dependent function-generator synthesis method. The authors introduce the concept of "interval reliability synthesis". The dimensions of the link lengths are treated as random variables while their mean values become the design variables, and the probability of failure to produce the function within a prescribed tolerance is minimised over a defined time interval and corresponding position level interval of the function. While these methods achieve excellent results, they do not shed any light on the curious tendency observed in Ref. [2]. What the vast body of literature reporting investigations into function-generator synthesis optimisation is missing is a systematic study of what the implications are of allowing the cardinal number of the I/O data set to tend towards infinity.

Hence, the goal of this paper is to take the first step towards proving that the convergence observed in Ref. [2] is true for planar four-bar function-generators. More precisely, a proof will be given for the design error that as the cardinality of the I/O data set increases from discrete numbers of I/O pairs to an infinite number between minimum and maximum pairs that a lower bound for any p-norm of the design error exists, and corresponds to that of the infinite I/O set, thereby changing the discrete approximate synthesis problem to a continuous approximate synthesis problem. To this end, the design error minimisation occurs in the space of a continuous function possessing an L_p norm defined later in this paper. However, our study is currently restricted to the planar RRRR function-generating linkage, where R denotes *revolute joint*, synthesised using the kinematic model defined in Ref. [10].

2. Design error minimisation: the discrete approximate approach

The synthesis problem of planar four-bar function-generators consists of determining all relevant design parameters such that the mechanism can produce a prescribed finite set of m I/O pairs, $\{\psi_i, \varphi_i\}_1^m$, where ψ_i and φ_i represent the i^{th} input and output variables, respectively, and m is the cardinality of the finite data-set. We define n to be the number of independent design parameters required to fully characterise the mechanism. For planar RRRR linkages, $n = 3$ [10]. If $m = n$, the problem is termed *exact synthesis* and may be considered a special case of approximate synthesis where $m > n$.

We consider the optimisation problem of planar four-bar function-generators as the approximate solution of an overdetermined linear system of equations with the least error. The synthesis equations that are used to establish the linear system for a four-bar function-generator are the *Freudenstein equations* [10]. Consider the mechanism in Fig. 1. The i^{th} configuration is governed by:

$$k_1 + k_2 \cos(\varphi_i) - k_3 \cos(\psi_i) = \cos(\psi_i - \varphi_i), \tag{1}$$

where the k's are the *Freudenstein parameters* , which are the following link length ratios:

$$k_1 = \frac{(a_1^2 + a_2^2 + a_4^2 - a_3^2)}{2a_2a_4}; \quad k_2 = \frac{a_1}{a_2}; \quad k_3 = \frac{a_1}{a_4}. \tag{2}$$

Given a set of three Freudenstein parameters, the corresponding set of link lengths, scaled by a_1, are:

$$a_1 = 1; \quad a_2 = \frac{1}{k_2}; \quad a_4 = \frac{1}{k_3}; \quad a_3 = (1 + a_2^2 + a_4^2 - 2a_2a_4k_1)^{1/2}. \tag{3}$$

The finite set of I/O equations can be written in the following form, using Eq. (1)

$$\mathbf{Sk} = \mathbf{b}, \tag{4}$$