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Development-process-of-nonlinearity-based reliability evaluation of structures

Jian-Bing Chen[1], Jie Li*

School of Civil Engineering, Tongji University 1239 Siping Road, Shanghai, PR China

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Abstract

A reliability evaluation approach based on the development process of the structural nonlinearity is presented. The traditional structural system reliability theory for structural safety regarding combination of failure modes is first revisited. It is seen that it stemmed from, and was heavily affected by, the assumption of perfect elasto-plasticity of materials. This will make the number of the failure modes increase in a non-polynomial form against the number of the potential plastic hinges. Moreover, the above methodology does not work appropriately in the case of nonlinearity in general form other than perfect elasto-plasticity, as commonly encountered in engineering practice. Discussions show that total information of the structure is involved in the development process of its nonlinearity, be it a deterministic case or stochastic counterpart. The information needed for reliability evaluation of structures could be extracted, for example, by capturing the probabilistic information of the extreme value of the corresponding response, which could be obtained by using the probability density evolution method. Therefore, the reliability evaluation for structural safety could then be directly evaluated without searching the failure modes. Taking a 10-bar truss as an example, the proposed method is theoretically elaborated and numerically exemplified.

Keywords: Reliability; Random variable; Nonlinearity; Failure mode; Probability density evolution method

1. Introduction

With the development of analytical and numerical methods in conjunction with computer techniques, reliability evaluation has been playing a role of increasing significance as one of the most important issues in structural engineering [1,2].

For the convenience of research and application, structural reliability is usually divided into two levels: the reliability at element level and the reliability at system level [3]. For the latter, there are usually a large number of failure modes (also referred to as non-rigid systems or mechanisms) which must be taken into account in the system reliability evaluation. This makes it difficult, or almost impossible, to obtain an exact solution of the system reliability for large structures. Therefore, a variety of approximate approaches have been developed. Among those approaches, two major, difficult problems are: (a) the number of the failure modes increases in a non-polynomial form against the number of the potential plastic sections or plastic hinges; and (b) how the correlation information among different failure modes, which is needed in computation of the probability of failure, can be captured and dealt with. The efforts devoted to the first problem have resulted in the development of approaches for identifying significant mechanisms and using the lower-upper bound approach (for example, [1,4–8], etc.). These explorations were undoubtedly fruitful and have led to appreciable progress. Nonetheless, being an intrinsic NP-hard problem, it is still too early to say, at present, that this family of approaches is promising in the near future to deal with the reliability evaluation of large structures with acceptable accuracy and efficiency. For the second problem, concerning correlation information among different failure modes, research results are relatively scarce. In most cases the correlative coefficients needed in the computation are assumed empirically when they are unavailable [9,10]. However, it is interesting and remarkable that the above two

* Corresponding author. Tel.: +86 21 65983526; fax: +86 21 65986345.
E-mail addresses: Chenjb@mail.tongji.edu.cn (J.-B. Chen), Lijie@mail.tongji.edu.cn (J. Li).
[1] Tel.: +86 21 65981505; fax: +86 21 65986345.

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problems will not occur in the Monte Carlo simulation method regardless of the requirement for large sampling numbers. This is an obvious advantage and consequently, endeavors have been devoted to improvements of the Monte Carlo simulation, for instance, the importance sampling and different variance reduction techniques (for example, [11,12]).

The traditional structural system reliability theory employs the concept of mechanism or failure modes, which comes originally from the elasto-plastic analysis of the structures [13]. The theorem of invariance of collapse loads strengthened the theoretical basis to some degree, or so was the researchers' belief. However, the methodology is essentially a phenomenological treatment on structural safety based on the failure consequences. This makes it necessary to define structural failure as a combination of mechanisms (failure modes). In the analysis, the development process of the structural nonlinearity is out of consideration and therefore the physical mechanism of the structural failure is not taken into account. In order to evaluate structural reliability comprehensively, what leads to structural failure, and how it occurs, should be studied; i.e., the reason for, and the physical mechanism of, structural failure. Therefore, the development process of the structural nonlinearity ought to be considered. The structural reliability evaluation could thus be viewed from the new angle of the development process of the structural nonlinearity instead of the traditional phenomenological based failure modes. In this way, the reliability evaluation becomes a problem of how to extract appropriate information from the development process of the structural nonlinearity, which could be realized, for example, through capturing corresponding extreme values when the probability density evolution method is employed [14,15]. In the present paper, the above thoughts are elaborated taking a 10-bar truss as an example. It is seen that when viewed from the development process of the structural nonlinearity instead of the failure modes, the main difficulties encountered in the system reliability evaluation can be avoided.

2. Development process of nonlinearity and limit states/failure modes

2.1. Assumption of perfect elasto-plasticity and failure modes

In modeling nonlinear behaviors of materials and structures, plasticity of materials was first studied by researchers. To make the plastic analysis of structures analytically or numerically tractable with bearable computational efforts, the perfect elasto-plastic or rigid-plastic model was used as the constitutive model of the materials. This led to the method of mechanism which has dominated the field of plastic analysis of structures for decades [16,13]. The above background had influenced the early developments of the structural system reliability either directly or indirectly [2].

To give clear insight to the cases, a 10-bar truss, one of the classical examples in the investigations of system reliability evaluation [6], as shown in Fig. 1 will be elaborated. The truss is subjected to a horizontal load F_1 on the joint 3, geometric sizes of the truss read $l_0 = 1.0$ m, $h = 1.5$ m. The sections

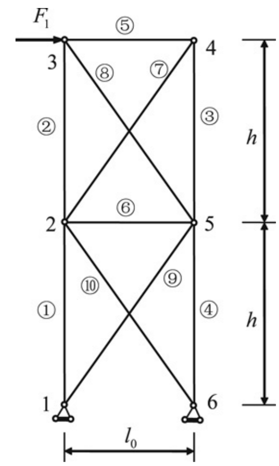

Fig. 1. The 10-bar truss.

of the bars have an identical area of 0.0001 m^2 with the initial elastic modulus of steel $E = 2.06 \times 10^{12}$ Pa. The compressive strength of the bars is half of the corresponding tensile strength. Perfect elasto-plastic model for the stress–strain relationship is employed as shown in Fig. 2(a), where σ_y and ε_y are the yielding stress and yielding strain, respectively.

The bearing capacity of the truss could be evaluated through the equilibrium of the limit states corresponding to different mechanisms. According to the upper-bound theorem of limit analysis [13], the computed capacity is the upper-bound of the real capacity wherein employing the real mechanism will yield the infimum which is the real capacity. In other words, if we can enumerate all possible non-rigid systems of the truss, we can compute the capacity corresponding to each mechanism by the equilibrium equation or principle of virtual displacement and then get the infimum one. For instance, three typical mechanisms are shown in Fig. 3, where the yielding bars are replaced by a pair of forces equal to the strength of the bars. Considering the equilibrium of the mechanism with additional applied forces replacing the yielded bars, one can get the following equations respectively for the mechanisms shown in Fig. 2(a), (b) and (c),

$$F_1 \cdot 2h = C_4 l_0 + C_{10} h \cos\varphi \tag{1}$$

$$F_1 = T_9 \cos\varphi + C_{10} \cos\varphi \tag{2}$$

$$F_1 = T_7 \cos\varphi + C_8 \cos\varphi \tag{3}$$

Here, C_j, T_j $(j = 1, 2, \ldots, 10)$ denote the compressive and tensile strength of the jth bar, respectively, $\cos\varphi = 2/\sqrt{13}$.

It is seen from Table 1 that Eqs. (1) through (3) will yield different upper-bounds wherein the real bearing capacity corresponding to different sets of parameters is the infimum because one of the mechanisms is the real mechanism (non-rigid system), which will be interpreted later in more detail.

The above analysis indicates that once the possible mechanisms are enumerated, the bearing capacity could be evaluated without concerning the development process of the structural nonlinearity. This appears to be a great advantage considering that it is much easier to solve the equilibrium equations of the limit states (say, Eqs. (1)–(3)) than to carry