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Adaptive Monte Carlo and GUM methods for the evaluation of measurement uncertainty of cylindricity error

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A R T I C L E I N F O

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A B S T R A C T

Measurement uncertainty is one of the most important concepts in geometrical product specification (GPS). The "Guide to the expression of uncertainty in measurement (GUM)" is the internationally accepted master document for the evaluation of uncertainty. The GUM method (GUMM) requires the use of a first-order Taylor series expansion for propagating uncertainties. However, when the mathematical model of measurand is strongly non-linear the use of this linear approximation may be inadequate. Supplement 1 to GUM (GUM S1) has recently been proposed based on the basis of probability density functions (PDFs) using the Monte Carlo method (MCM). In order to solve the problem that the number of Monte Carlo trials needs to be selected priori, adaptive Monte Carlo method (AMCM) described in GUM S1 is recommended to control over the quality of the numerical results provided by MCM.

The measurement and evaluation of cylindricity errors are essential to ensure proper assembly and good performance. In this paper, the mathematical model of cylindricity error based on the minimum zone condition is established and a quasi particle swarm optimization algorithm (QPSO) is proposed for searching the cylindricity error. Because the model is non-linear, it is necessary to verify whether GUMM is valid for the evaluation of measurement uncertainty of cylindricity error. Then, AMCM and GUMM are developed to estimate the uncertainty. The procedure of AMCM scheme and the validation of GUMM using AMCM are given in detail. Practical example is illustrated and the result shows that GUMM is not completely valid for high-precision evaluation of the measurement uncertainty of cylindricity error if only the first-order terms in the Taylor series approximation are taken into account. Compared with conventional methods, not only the proposed QPSO method can search the minimum zone cylindricity error precisely and rapidly, but also the Monte Carlo simulation is adaptive and AMCM can provide control variables (i.e. expected value, standard uncertainty and lower and higher coverage interval endpoints) with an expected numerical tolerance. The methods can be extended to the evaluation of measurement uncertainty of other form errors such as roundness and sphericity errors.

1. Introduction

All measurement processes have uncertainty to some extent. When a measurement result is reported, it is necessary to give the uncertainty associated with the measurement. The GUMM provides a framework for assessing uncertainty based on the law of propagation of uncertainty and the characterization of the output quantity by a Gaussian distribution or a scaled and shifted *t*-distribution. GUMM requires the use of a first-order Taylor series expansion for propagating uncertainties. When the mathematical model of measurand is strongly non-linear the use of this linear approximation may be inadequate. Since GUMM has some limitations [1,2], GUM S1 has recently been proposed by the Joint Committee for Guides in Metrology (JCGM) [3] that is implemented by MCM. It is recommended that both GUMM and MCM are applied for non-linear model and the results are compared [4].

The cylindrical feature is one of the most basic geometric primitives which contribute significantly to fundamental mechanical products such as revolving devices, assembly parts, injection molds, transmission systems and precision gauges to achieve intended functionalities. The measurement and evaluation of cylindricity errors are essential to ensure proper assembly and good performance. In order to obtain a reliable assessment of the cylindrical form, an appropriate extraction strategy for obtaining a representative set of points on the workpiece is required. For determining an appropriate strategy, the harmonic content of the workpiece in both the roundness and generatrix directions is of prime importance [5]. This will determine the theoretical minimum density of points to cover the workpiece. In practice, it is often difficult to achieve a complete covering of the cylindrical feature given

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by the theoretical minimum density of points. In these situations more limited extraction strategies are employed which give specific rather than general information concerning the assessment of the cylindrical form. The strategies include bird-cage extraction, roundness profile extraction, generatrix extraction and points extraction strategies [5]. Coordinate measuring machines (CMMs) are basic instruments in modern metrology and are used extensively for carrying out online and offline inspection with less measurement uncertainty. To yield critical geometric deviations of the measured parts, data obtained by CMMs must be analyzed or interpreted using appropriate algorithms, which should conform to the specifications. Current CMMs verification algorithms are based on the least square method (LSM) because of its ease of computation and the uniqueness of its solution for linear problems as well as its unbiasedness of its solution for uncorrelated Gaussian noise. However, the LSM does not guarantee the minimum zone solution (MZS) specified in the ISO/1101 standard [6]. Therefore, the minimum zone methods (MZM) of cylindricity error evaluation have received much attention in recent years.

Carr and Ferreira [7] formulated cylindricity and straightness of a median line as non-linear problems, which were then transformed into a series of linear problems that converged to the non-linear solution points in the set. Lai and Chen [8] employed a non-linear transformation to convert a circle into a line and a cylinder into a plane. Then a straightness or flatness evaluation scheme was employed to obtain control points and the minimum zone deviation for the feature parameters and a series of inverse transformation procedures was then implemented to compute desired feature parameters. Chou and Sun [9] developed supplementary methodologies for finding a fine-tuned axis of the cylinder, including rotational devices, quasi-linear equations and complicated procedures. Zhu and Ding [10] applied kinematic geometry to calculate the cylindricity error. Shunmugam and Venkaiah [11] developed the algorithms based on computational geometric concepts to arrive at the minimum circumscribed, the maximum inscribed and the minimum zone circular cylinders for solving cylindricity errors. With the emergence of computational intelligence, the intelligence-oriented such as genetic algorithms (GAs) [12], particle swarm optimization algorithm (PSO) [13] and a hybrid particle swarm optimization–differential evolution algorithm [14] were employed to evaluate cylindricity errors. However, the methods above were only used to compute cylindricity errors and could not estimate the measurement uncertainty. Recently, Mao et al. [15] proposed to calculate the minimum zone cylindricity error using PSO and developed an uncertainty evaluation method based on GUMM. Although GUM is the de facto standard for the evaluation of the measurement uncertainty in metrology, it is mainly applicable to the linear models. GUM S1 provides a general numerical approach implemented by MCM and it is consistent with the broad principles of GUM for carrying out the calculations required as part of an evaluation of measurement uncertainty [3]. It also provides guidance in situations where the conditions for GUMM are not fulfilled, or it is unclear whether they are fulfilled [3]. Therefore, recently some researchers have developed MCM to evaluate measurement uncertainty. Matus [16] employed MCM to evaluate the measurement uncertainty of a special form deviation of gauge blocks whereas GUMM cannot be applied to this problem, because the partial derivatives of the model function do not exist. Moschioni et al. [17] proposed a method that combined the factorial design of experiments and MCM to guide the instrument designer in the instrument configuration optimization. Kruth et al. [18] presented a method to determine measurement uncertainties for feature measurements on CMMs based on Monte Carlo simulations and a profile database of realistic form profiles. Lian and Chen [19] proposed the uncertainty evaluation of roundness measurement based on MCM, but the simulation trials need to be set in advance. Decker et al. [20] applied MCM to the evaluation of measurement uncertainty for grating pitch calibration by optical diffraction. Wübbeler et al. [21] drew a conclusion that MCM was a reliable tool and generally the method of choice if the GUM uncertainty framework was not adequate and it could work with any description of a model (not necessarily in explicit mathematical form) as well as it could treat input quantities that were not independent. Andolfatto et al. [22] proposed a method to evaluate contributions to the uncertainties for identified link errors of a five axis machine tool. And those standard uncertainties were propagated with a multi-output adaptive Monte Carlo approach, using either a statistical model or a cyclic model for the drift. Theodorou et al. [23] used both MCM and GUM Uncertainty Framework to estimate the uncertainty of the direct determination of cadmium in water by graphite furnace atomic absorption spectrometry. The results showed that the GUM uncertainty framework slightly overestimates the overall uncertainty by 10%. The main source of this difference is the approximation used by the GUM uncertainty framework in estimating the standard uncertainty of the calibration curve produced by least squares regression. Meyer et al. [24] presented a detailed comparative study between the GUMM and the MCM for the uncertainty evaluation of the yield of streak cameras used on the Laser Integration Line facility to record highly transient physical processes taking place within laser-produced plasmas.

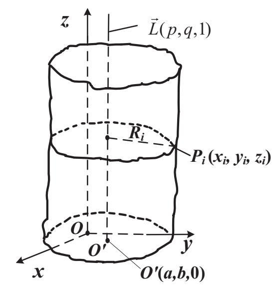

Fig. 1. A cylinder in measurement coordinate system.

Considering that the mathematical model of the minimum zone cylindricity error is non-linear, it is necessary to verify whether GUMM is valid for the evaluation of measurement uncertainty of cylindricity error. AMCM and GUMM are developed to estimate the measurement uncertainty and the results are compared. The paper is organized as follows: the model of cylindricity error based on the minimum zone association method is established. Then, QPSO is proposed to calculate the minimum zone cylindricity error. AMCM and GUMM for the evaluation of measurement uncertainty of cylindricity error are developed. Finally, practical example is illustrated and conclusions are summarized.

2. Mathematical model of the minimum zone cylindricity error

A cylinder in three dimension coordinate system *oxyz* is shown in Fig. 1. Assuming P_i (x_i, y_i, z_i) (i = 1, 2, ..., N) are the measured points of the cross-section profile obtained by measuring a cylindrical part. The axis L and coordinate axis z are parallel and O'(a, b, 0) is the point of intersection between the axis L and coordinate plane *xoy*. L can be expressed as:

$$\frac{x-a}{p} = \frac{y-b}{q} = z \tag{1}$$

where a, b and 0 are the coordinates of point O' in the coordinate system *oxyz*, p, q and 1 are the components of the directional vector