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Dynamic analysis of mechanical systems with planar revolute joints with clearance

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ABSTRACT

Clearances in mechanical joints are unavoidable due to many uncertainties such as manufacturing tolerances, assemblage, wear, and material deformation. The dynamic characteristics of mechanical systems are greatly affected by the joint clearances during the contact process. Thus, a proper contact force model plays a key role in simulating the overall performance of the mechanical systems. In this paper, a hybrid contact force model is described, which is based on the Lankarani–Nikravesh contact force model and the elastic foundation model. The discrete element theory and Gaussian quadrature are utilized to analyze and simulate the contact process. By distributing the non-uniform bearing points over the pin surface and calculating the penetration of each point individually, the total contact force can be obtained by integrating the discrete ones in the contact area. Some comparisons with the Lankarani–Nikravesh model indicate that the hybrid model is more effective for small clearance and low restitution coefficient situations. Finally, comparison of dynamic characteristics between the numerical simulation results and experimental ones for a revolute clearance joint in the slider–crank mechanism is employed to validate the hybrid model.

1. Introduction

It is well known that the mechanical components are linked by joints in which clearances are always present. The existence of clearances in joints will induce vibrations and noises which will affect the transfer of the system load and lead to poor performance of the mechanical systems [1–3], especially for high speed situations. Thus, a proper contact force model for quantifying the contact effects on the entire system becomes very important [4–7]. More and more researchers have devoted themselves to investigating the treatment of clearances, and extensive work has been done to study the dynamic responses of joints with clearances in multibody systems.

By and large, an impact process may be divided into two phases, namely, the loading phase and the unloading one. During the loading phase, the two bodies move close to each other and deform in the normal direction of the contact plane. The relative normal velocity of the contact points gradually decreases to zero when the maximum of the relative penetration is reached, which also represents the start of the unloading phase. During the unloading phase, the two bodies move away from each other as the relative penetration gradually declines to zero, and separate in the end.

In general, there exist two different methods for impact analysis in multibody systems, that is, the discontinuous method and the continuous one [8,9]. The discontinuous method assumes that the period of the impact is very short and the configuration of the system doesn't change. The contact process is divided into two stages—before and after the impact, and they are linked by the law of momentum conservation [8,9]. Due to the assumption of the short impact process, the velocities present instantaneous changes,

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thus the accelerations are infinite. The restitution coefficient is introduced to quantify the energy transfer, which equals one for purely elastic contacts, and zero for purely plastic contacts [10,11]. It is indicated that many factors may have effects on the selection of the restitution coefficient, for example, material properties, geometry configuration, and the relative speed of the contacting bodies. Valuable work on restitution coefficient has been done by Johnson and Thornton [12,13]. Although the discontinuous method has been utilized by many researchers, the unknown duration of the impact process limits its application. If the duration time of the impact is so long that great changes take place in the structure of the system, the assumption of the short period becomes invalid [8–10]. The continuous method, which is in fact a penalty method, assumes that the forces and penetrations vary continuously. The contact force is applied in the direction perpendicular to the plane of the contact, and different models have been put forward to explain the changes of the forces in the contacting surface [10]. The simplest one is known as the Kelvin–Voigt model, which is established by a linear spring-damper element [14]. In this model, the spring represents the elastic behavior of the contacting bodies while the damper describes the energy dissipation during the contact process. In general, both the stiffness and damping coefficients are assumed to be known. However, this model cannot give an accurate description of the overall nonlinear nature of the contact process due to its simplicity [10,15]. The best known nonlinear force model for the contact between two spheres of isotropic materials was firstly developed by Hertz [16]. As Hertz force model does not account for the energy dissipation during the contact process, it cannot model the whole contact process accurately. Consequently, a great number of contact force models were proposed that augmented the Hertz force model with a damping element to accommodate the energy dissipation during the contact process for small or moderate impact velocities [17]. Hunt and Crossley [18] held that the damping coefficient should be proportional to the elastic force in the contact process and they put forward a contact force model with a nonlinear viscoelastic element, which has been used by many researchers for its simplicity [19–21]. Also, Herbert and McWhannell [22] presented a force model which could be considered as a refinement of [18]. Lee and Wang [23] proposed a force model, similar to [18], focusing on the satisfaction of the expected hysteresis boundary conditions, namely, zero damping force at zero and maximum relative penetrations of contact. A widely used contact force model with hysteresis damping element was proposed by Lankarani and Nikravesh [8], which has been utilized by numerous authors for different problems [5,24–27]. A complete and systematic comparison among different contact force models has been made by Machado et al., which can provide information on their application range and accuracy in different contact situations [17]. And it must be highlighted that the contact force models described above are somewhat limited as they do not work adequately for low restitution coefficients [17]. To overcome this problem, Gonthier et al. [28] developed a volumetric contact force model for multibody dynamics. For a purely inelastic contact, the hysteresis factor in this model is infinite, which means it can perform well for perfectly inelastic contact. Qin and Lu [29] described a contact force model which presented a superior response mainly for low values of the restitution coefficient [17]. Recently, another continuous contact force model proposed by Flores and his colleagues can be applied in the entire range of the restitution coefficient, that is, in the cases of soft and hard contacts [30].

For the purpose of calculating the local deformations, Hertz [16] introduced the simplification that each body can be taken as an elastic half-space loaded over a small elliptical region of its plane surface, which can be met only in the case that the clearance between two contacting bodies is large enough and the load is small [1,12]. Unfortunately, clearances existing in actual joints of mechanical systems are usually very small, so the results obtained by Hertz model and its modified versions are usually not reliable. Moreover, the contact stiffness and damping coefficients calculated in most literatures up to date rely on the radii of the contacting bodies and the research is mainly restricted to simple geometries like spheres and cylinders.

In this paper, a hybrid contact force model is presented, along with a hysteresis damping element which accounts for energy dissipation during the entire contact process. The frame of the proposed model is similar to that of Lankarani–Nikravesh contact force model, and the contact stiffness coefficient is based on the elasticity theory when the effect of the transverse deformations is neglected in the calculation of the normal traction, which has been used in the elastic foundation model [31–33]. Moreover, the damping coefficient modified by Qin and Lu [29] is employed in this paper. Enlightened by the discrete element theory, the pin surface between the two contacting bodies is divided into elements instead of a whole body for contact analysis in this paper. In addition, the Gaussian quadrature [34] enables to distribute bearing points on each element unevenly. Penetration of each bearing point needs to be calculated individually at each time step to judge the occurrence of contact. Then, the total contact force can be obtained by integrating the discrete one in the contact area. The tangential contact effect is evaluated by using the Ambrósio friction law [2,10]. The methodology and procedures adopted throughout this work are presented and analyzed with the help of a numerical simulation of the classical slider–crank mechanism with revolute joint clearance problem.

2. Equations of motion for constrained multibody systems

A multibody system is a collection of several components connected by different joints which constrain their relative motions by imposing forces and moments. The formulation of multibody system dynamics used in this work keeps consistent with that of Nikravesh [35], in which the system configuration is described by the generalized Cartesian coordinates [2]. The solution of the problem for constrained multibody systems can be obtained by using the method of Lagrange equation which leads to a set of differential and algebraic equations (DAE), in which the coordinates and the Lagrange multipliers are unknown quantities. To reduce the numerical stability problems, the Baumgarte stabilization technique is employed [36,37].

For a constrained multibody system, the algebraic kinematic independent holonomic constraints Φ can be expressed as [38–40],

$$\Phi(\mathbf{q}, t) = \mathbf{0} \tag{1}$$