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A low-frequency pendulum mechanism

Uwe Starossek *

Hamburg University of Technology, Denickestr. 17, 21073 Hamburg, Germany

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ABSTRACT

A pendulum mechanism is presented whose natural frequency of oscillation is distinctly lower than that of a conventional pendulum of comparable size. Furthermore, its natural frequency is approximately proportional to its amplitude of oscillation. The mechanism can thus be tuned to extremely low frequencies by using small amplitudes. The undamped free oscillation response of the mechanism is studied. The derivation of the equation of motion is outlined for both large and, after neglecting higher order terms, small displacements. In both cases, a second-order nonlinear differential equation results. When higher order terms are neglected, the equation of motion is of simple form and can be solved symbolically in terms of a Jacobi elliptic function. Based on this solution, a closed-form expression for the natural frequency is derived and the characteristics of the free oscillation response are discussed.

1. Introduction

The development of the mechanism studied here was inspired by the search for low-frequency pendulum mechanisms. Such mechanisms have applications in engineering and science. They can be used, for instance, as tuned mass dampers for large engineering structures, which have low natural frequencies of vibration. Tuning a conventional pendulum to a low frequency requires a correspondingly large rod length, which not always can be accommodated.

The mechanism is shown in Fig. 1. It consists of a massless rod or cord of length H suspended from a pivot at its upper end, a massless rigid beam hinged to the lower end of the rod, a frictionless sliding horizontal support at a certain point of the beam, and two identical point masses $m_1 = m_2 = m / 2$. The masses are attached to the beam on opposite sides of, and at the same distance L, from the lower hinge (i.e., the hinge at the lower end of the rod). The distance along the beam between the lower hinge and the horizontal support is called l. Note that l can also be equal to or larger than L. The gravitational acceleration is denoted by g. Horizontal, in the foregoing sentences, means perpendicular to the direction of gravity. Only displacements in the drawing plane are considered.

The mechanism has one degree of freedom. Fig. 1 shows the static equilibrium position, in which the rod is oriented vertically and the beam horizontally. Fig. 2 shows the mechanism in a displaced position, where the beam has moved vertically at the sliding horizontal support but not horizontally. After inducing a displacement, the beam oscillates about its equilibrium position in a combined rotational and translational motion, whereas the rod, due to the kinematic constraints, swings to only one side of its equilibrium position. The masses move in mainly vertical and opposite directions. The swinging motion of the rod goes along with a periodic lifting of the center of gravity. Both these motions are small in relation to the motions of the individual masses. Hence the restoring forces, compared to a conventional pendulum, are likewise small in relation to the inertia forces and, thus, the natural frequency of oscillation will be comparatively low.

* Tel.: +49 40 42878 3987.
E-mail address: starossek@tuhh.de.

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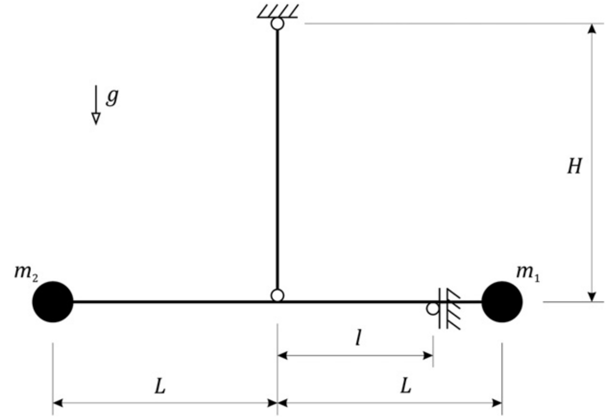

Fig. 1. Static equilibrium position of pendulum mechanism.

In the following, the derivation of the equation of motion for the undamped free oscillation response of the mechanism is first outlined for large displacements. The resulting second-order nonlinear differential equation is too complicated to be solved symbolically and could only be solved numerically. Instead, the derivation is simplified by neglecting higher order terms, which limits the validity of the results to small displacements. A second-order nonlinear differential equation consisting of two simple terms is obtained.

An exact symbolic solution of this equation is derived in terms of a Jacobi elliptic function. The resulting free oscillation response time histories are presented and discussed, and a closed-form expression for the natural frequency of oscillation is derived. It is found that the natural frequency is proportional to the amplitude of oscillation and, as expected, that it is much lower than that of a conventional pendulum of comparable size.

2. Equation of motion

The displacement components of the lower hinge in the horizontal and vertical directions, related to the static equilibrium position, are denoted by x and y, respectively (see Fig. 2). The rotational displacement of the beam is denoted by α and that of the rod by β. Because the mechanism has only one degree of freedom, these four displacement variables are kinematically related. Nonetheless, their status is different given that, among these four, only α uniquely defines the state of the system and can serve as an independent variable. Variables x, y, and β can assume only positive values such that for any given value of x, y, or β two values of α exist, one positive and one negative. An alternative independent displacement variable is

$$v = l \sin \alpha. \tag{1}$$

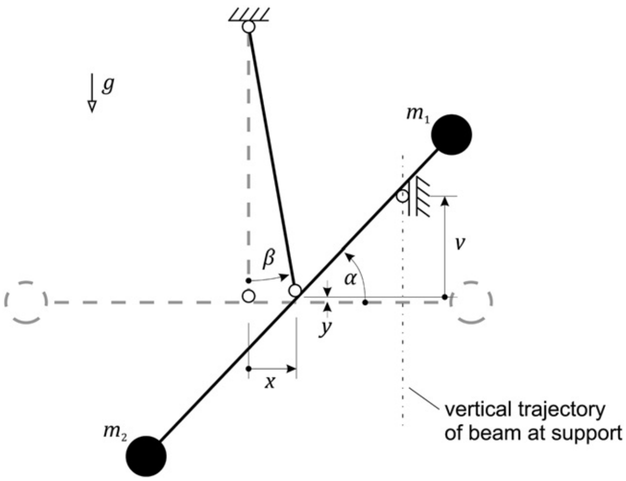

Fig. 2. Pendulum mechanism in displaced position.