Mechanism and Machine Theory 83 (2015) 91–108

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

A new geometric meshing theory for a closed-form vector representation of the face-milled generated gear tooth surface and its curvature analysis

Yuansheng Zhou, Zezhong C. Chen *

Department of Mechanical and Industrial Engineering, Concordia University, Montreal H3G1M8, Canada

ARTICLE INFO

Article history:
Received 5 June 2014
Received in revised form 21 August 2014
Accepted 3 September 2014
Available online 3 October 2014

Keywords:
Geometric meshing theory
Equation of meshing
Closed-form vector representation
Envelope surface
Generated gear
Curvature analysis

ABSTRACT

The tooth surface and its curvature are fundamental inputs to evaluate the contact and transmission of spiral bevel gears or hypoid gears. Currently, the tooth surface of the face-milled generated gear is represented as an implicit form of three parameters, and one of which can be eliminated according to the well-known equation of meshing. Unfortunately, it is not easy to solve the equation of meshing due to the complex process of gear generation. Moreover, it is complicated to calculate the derivatives of the equation of meshing, and this makes it inefficient to use the results of well-established differential geometry to conduct curvature analysis. To address this problem, a new geometric meshing theory is proposed to obtain a closed-form (explicit) vector representation of the tooth surface, and curvature analysis is directly implemented with differential geometry equations. As a consequence, the calculation is straightforward and efficient. The example of a face-milled spiral bevel gear is presented as the application of the proposed method.

1. Introduction

Spiral bevel gears and hypoid gears are crucial to power transmission systems, such as the automobile, power generation machine and helicopter. In industry, the generating process of face-milled spiral bevel gears or hypoid gears can be classified as non-generated (Formate®) or generated methods, and the corresponding gears are non-generated gears or generated gears, respectively. Compared to the non-generated method, generating roll motions are applied for the generated method during the generating process. The non-generated method offers higher productivity than the generated method. However, the generated method provides more freedom to control the tooth surface and its curvature, which are significant to the contact and transmission of the gear drive. Theoretically, the tooth surface of the non-generated gear is a part of the head-cutter surface. In contrast, the tooth surface of the generated gear is more complicated, and it is a part of the envelope surface of the family of the head-cutter surfaces in the generating process. It is very challenging to calculate the generated gear tooth surface and its curvature. To better understand the background of such challenges, a detailed literature review is conducted below.

With the consideration of the gear generating process, the tooth surface can be represented as an implicit form of three parameters, and one of which has to be eliminated according to a necessary condition of the generating process. This necessary condition is written as a well-known equation, which is the equation of meshing [1,2]. According to the equation of meshing, tooth surface and its curvature analysis can be implemented with different methods. These methods can be categorized into two groups, Litvin's and invariant approaches, depending on whether the derivation process is related with coordinate systems or not. In the following

* Corresponding author.
E-mail address: zcchen@encs.concordia.ca (Z.C. Chen).

http://dx.doi.org/10.1016/j.mechmachtheory.2014.09.002

literature review, the tooth surface generation is introduced separately with both approaches. Subsequently, the curvature analysis based on both approaches is also illustrated respectively.

With the equation of meshing, the now classic Litvin's approach has been widely applied to calculate the tooth surface of the generated gear [1–5]. The calculation is conducted with the kinematic chain which is related to different coordinate systems. Based on the equation of meshing, Fong and Tsay [6] proposed a mathematical model for the tooth surface of circular-cut spiral bevel gears. Tsay and Lin [7] introduced a mathematical model for a universal hypoid gear generator. Shih et al. [8] developed the mathematical model for face-hobbed gears. The similar model proposed by Lelkes et al. [9] is specifically applied to Klingelnberg bevel gears. Vimercati [10] proposed accurate geometrical representation for the tooth surface of face-hobbed hypoid gears. Chen et al. [11] obtained a gear model by introducing an instantaneous screw axis to describe the generalized motion of gear generation.

Different from Litvin's approach, the invariant approach describes the gear generating process without referring to coordinate systems, and this makes the overall formulation compact. Two significant issues of the invariant approach are vector (or tensor) expression and motion description. Wu and Luo [12] used vector expression to formulate gear generation, and Dooner [13] introduced screw theory to describe the kinematic motion. Puccio et al. [14] represented gear generation based on geometric relations in vectorial form, and a more general case [15] of the gear generation with supplemental motion is introduced subsequently. Wang and Zhang [16] extended the invariant approach with tensor expression.

With the tooth surface model, curvature analysis is investigated to improve the contact and transmission of the gear drive. Mathematically, curvature analysis can be implemented with the results of classic differential geometry [17]. However, since the tooth surface of the generated gear is expressed as an implicit form with the equation of meshing, it is complicated and inefficient to calculate the derivatives of the tooth surface with differential geometry approach. Subsequently, the alternative approaches are proposed to curvature analysis. Litvin et al. [1–5] proposed a theory of gearing to do curvature analysis for conjugated surfaces. In this now classic approach, a systematic methodology is established to derive the curvature relationship between two conjugate surfaces based on the analysis of the motion of the contact point. This method employs kinematic relationships and scalar components in an orthogonal reference frame, and it does not use the parametric coordinates.

Based on Litvin's approach, Chen [18] introduced non-principal parametric coordinates and developed curvature expression in a given direction in a non-orthogonal reference frame. The case with the generalized motion has been developed by Chen et al. [11]. Wu and Luo [12] obtained the curvature equations by introducing screw theory to describe the relative motion of conjugate surfaces, and Yan and Cheng [19] applied this approach to some cam-follower mechanisms. Ito and Takahashi [20] investigated curvatures in hypoid gears with classic differential geometry and kinematic relationships. Dooner [13] proposedthe third law of gearing with screw theory to obtain the limiting relationship between the curvatures of two conjugate surfaces. Puccio et al. [21] used the invariant approach to do curvature analysis of conjugate surfaces. An extension of the invariant approach to the general case of the gear generation with supplemental motions is introduced consequently [15]. Puccio et al. [22] gave a comprehensive comparison in different methods and expressed all these methods with a vector form. Wang and Zhang [16] developed the invariant approach with rotation and curvature tensors to illustrate the theory of gearing and local synthesis.

All of the above research calculates the tooth surface based on the equation of meshing, and the derivatives of the equation of meshing are used to investigate curvature analysis with alternative approaches. However, the whole calculation process can bemore simple and straightforward. A geometric meshing theory, which comprises a geometric equation and a simplified equation of meshing, is proposed in this paper to obtain a closed-form vector representation of the face-milled generated gear tooth surface. Subsequently, curvature analysis can be implemented directly and efficiently with differential geometry equations. This paper is organized in seven sections. In Section 2, the equation of meshing is illustrated in detail. In Section 3, the geometric equation is obtained from a geometric characteristic of the surface of revolution. In Section 4, the simplified equation of meshing is derived by introducing the vector representation of the surface of revolution to the equation of meshing, and subsequently the geometric meshing theory is obtained by combining the geometric equation and the simplified equation. In Section 5, the geometric meshing theory is solved to obtain the closed-form vector representation of the face-milled generated gear tooth surface. In Section 6, curvature analysis is investigated with classic differential geometry equations. In Section 7, the example of a face-milled spiral bevel gear is presented to demonstrate the convenience and efficiency of the proposed method.

2. Equation of meshing

2.1. An introduction of the equation of meshing

The generating process is related to a generating surface continuously moving in the three dimensional Euclidean space $\mathbb{E}^3$. During the generating process, a family of surfaces is formed with respect to all configurations of the generating surface at every moment. The envelope surface of this family of the generating surfaces is the generated surface, which can be calculated according to the equation of meshing [1,2].

As shown in Fig. 1, the generating surface is a conical surface $\mathbf{r}(h, \theta)$, in which h is the parameter of generatrix and θ is the parameter of rotation. Suppose that the conical surface undergoes a general motion from the initial configuration to a new configuration during the generating process. Assume that the parameter of motion is ϕ, then the family of the conical surfaces can be represented as $\mathbf{r}(h, \theta, \phi)$ in $\mathbb{E}^3$ with respect to a chosen fixed point $o^{(f)}$. At a given instance ϕ^*, the configuration of the generating surfaces can be described as $\mathbf{r}(h, \theta, \phi^*)$.

During the generating process, the generated surface and the generating surface stay in line contact at every moment, this line is called contact line or characteristic. For a given instance ϕ^*, the contact line can be represented by eliminating either h or θ. Taking the