Mechanism and Machine Theory 82 (2014) 17–32

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Adaptive grid-size finite element modeling of helical gear pairs

Marco Barbieri *, Antonio Zippo, Francesco Pellicano

Department of Engineering "Enzo Ferrari", University of Modena and Reggio Emilia, Via Vignolese 905/B, 41125 Modena, Italy

ARTICLE INFO

Article history:
Received 20 February 2014
Received in revised form 5 July 2014
Accepted 21 July 2014
Available online 16 August 2014

Keywords:
Helical gears
NURBS profile
Adaptive grid
FE analysis
Loaded TCA

ABSTRACT

A method for solving the contact problem for generic helical gear pairs (either external or internal) is described. Gear profiles are obtained by means of numerical simulation of the cutting process and an accurate description is provided in terms of NURBS curves or surfaces. A new method for finding the enveloping profiles for a generic tool (e.g. from a measured topography) is proposed. The minimum number of parameters needed to describe the profile in the presence of tip and root reliefs, helix modification and crowning is discussed. A reference structured grid is defined on the tooth, then refinement criteria are applied in order to obtain accurate solutions in terms of tooth deflection, contact pressure and fillet stress. The method allows to automatically perform a Loaded Tooth Contact Analysis (LTCA) starting from the design data of a gear pair. Results from the LTCA include contact pressure and contact pattern maps, as well as maximum fillet stress and fatigue strength. Combining several analyses within a mesh cycle, information about the Static Transmission Error (STE) and mesh stiffness is provided. The whole procedure has been implemented in a software called helical pair. A comparison with other approaches is given.

1. Introduction

International standards [1] prescribe two methods to design a helical gear pair: method A and method B. The method B consist in the application of empirical formulae, while the method A consist in computing stresses in the gear pair under nominal load conditions. This computation (method A) is usually performed by means of finite element analyses, nonetheless defining a suitable model for the analysis is not straightforward. The main issue is that the contact in gear pair is localized in a very small region, close to a line or to a point, so that a very refined grid is required. Moreover, when the contact is shared by more than one pair of teeth, the contact load depends upon the micro-geometry of the gear profile; this requires an accurate modeling of the actual profile. An accurate modeling and solution of the gear contact problem is especially important whenever profile reliefs or profile errors and misalignments are present [2–9].

The problem of describing the actual shape of the contacting surfaces has been faced by many authors. In 1999, Litvin et al. presented a paper in which the problem of computerized meshing simulation in helical gear pairs was considered [10]. A way to address this problem is finding the enveloping surface of the cutting tools described in terms of interpolating functions [11]. Describing the real geometry is still a problem of great interest, especially when complex gear profiles are to be manufactured by means of face milling [12].

In order to overcome the high computational effort required by a fine grid modeling of the whole gears, some approaches have been developed in the past. In 1999, Vijayakar [13] proposed a semi-analytical technique to solve the problem of two bodies in counterformal contact. The main idea behind his work was to describe the sub-surface stress in the contact region by means of Hertz theory and to match the corresponding displacements with those computed by FE analysis for the remaining part of the elastic

* Corresponding author.
E-mail addresses: mark@unimore.it (M. Barbieri), antonio.zippo@unimore.it (A. Zippo), francesco.pellicano@unimore.it (F. Pellicano).

http://dx.doi.org/10.1016/j.mechmachtheory.2014.07.009
0094-114X/

bodies in contact; in the present paper, this solution is considered for comparisons. In 2010, Huang et al. [14] described a comprehensive procedure for performing a static FE analysis of a gear pair for different contact positions within a mesh cycle. Since no grid refinement is introduced, their method is able to compute the stresses close to the fillet, but no information about the actual contact pressure is provided. The same issue can be found whenever approximated analytical models for the contact are used [15–18].

More recently, some solutions involving a refined grid close to the contact have been proposed [19–21]. These approaches allow to obtain accurate solutions both in terms of fillet stress and of contact stress; nonetheless, they kept a structured mesh in the refined case, so that some elements were over-stretched. Moreover, the method was not suitable in case of helical gears and/or misalignments, because in this case the contact region is no more coincident with the refined grid. In 2007, Mao [22] proposed a self-refining mesh model: in his solution the whole flank in contact is refined. This approach is still too much time consuming for solving the contact problem on highly refined grids.

In the present paper a method involving a standard FE solver is described. First of all the exact geometry of the gear profile is developed, since the contact problem is highly sensitive to it. Then, a computational grid is defined over the bodies in contact: the grid must be refined enough, close to the contact, so that the high stress gradients that are typical of the Hertzian solution can be accurately represented. In order to avoid refining the grid over the whole computational domain, an iterative refinement is used, both with structured and unstructured meshes. It is to point out that, in the case of low loads, the contact region can be extremely small with respect to the tooth surface; in this case the use of an adaptive grid is mandatory in order to achieve a good description of the contact.

The grid refinement requires transferring a high precision description of the gear surface to the finite element solver, in order to impose that the new superficial nodes are attached to the actual geometry by the automatic grid refinement; this causes an unacceptable growth of the data storage required, if a linear approximation is used. Therefore, Non Uniform Rational Basis Spline NURBS, which are de facto the standard method in the surface description, are used here to approximate the gear profile, both for pure involute profiles and for modified gears; the same NURBS define the profile of the trochoidal fillet as well. Using this approach, it is quite straightforward to perform the convergence analysis of the numerical solution. The accuracy of the method is checked by means of comparisons with the literature.

2. Profile description

Two kinds of geometries are given here: profiles obtained by hob cutting (in the form of the equivalent basic rack) and involute profiles with a circular fillet of given radius. For the first kind, the rack profile is discretized by means of a large number of points (some thousands) in order to have a very accurate profile. For each discretization point, the corresponding final profile point is computed solving the mesh equation

$$\overline{n}_1(\theta) \cdot \overline{v}_{12}(\theta, \phi) = 0 \tag{1}$$

where $\overline{n}_1(\theta)$ is the normal vector to the enveloping curve or surface, expressed as a function of the parameter θ, see Fig. 1; $\overline{v}_{12}$ is the relative velocity of the contacting profiles, ϕ is the parameter of the relative motion of the rack with respect to the manufactured gear. More details about the enveloping procedure can be found in [23]; here it is worthwhile to note that the mesh equation cannot be solved in a closed form for a generic rack shape: for this reason, in the present work, a Newton–Raphson procedure is used.

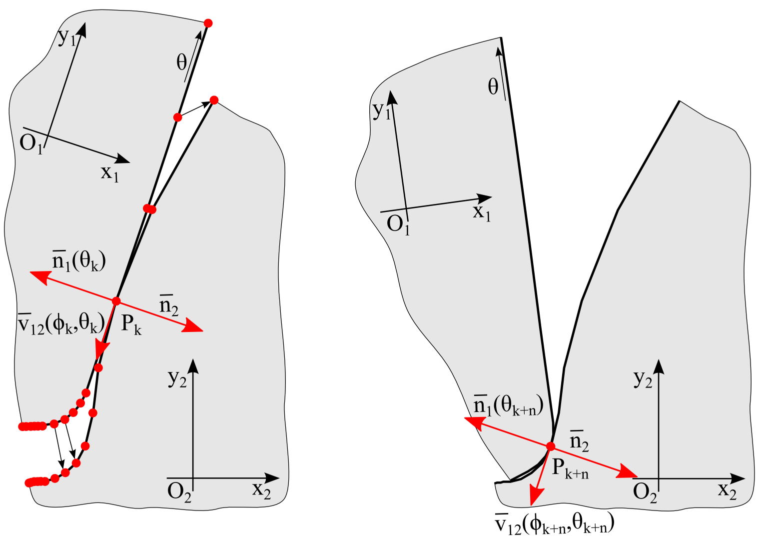

Fig. 1. Simulation of the enveloping process for a few points.