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Effects of gear mesh fluctuation and defaults on the dynamic behavior of two-stage straight bevel system

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ABSTRACT

This paper presents a three-dimensional model of two-stage straight bevel gear system. Periodic fluctuations of the gear meshes' stiffness are the main sources of excitation for the faultless gear system. First, dynamic response is calculated using Newmark method which is step-by-step time integration. The numerical results are presented in both the frequency and time domains. After that, we introduce some defects in the developed model such us the eccentricity defect, profile error and cracked tooth. The dynamic behavior of the defected gear system is compared with that of the faultless system. The numerical results from this work have applications in the design phase or maintenance of this type of bevel gear transmission.

1. Introduction

Gears are an excellent mechanism for power transmission due to their uses and works that are devoted. In addition to these benefits, bevel gear allows the transmission angle changing. Indeed, most studies are devoted to the dynamic behavior of a single stage gearbox. Merzouk [1] modeled one spur gear system by the finite element method concentrates. Yakhou [2] addressed the problems of gearboxes with helical teeth; he cited different sources of radiated noise of these boxes. Bruyere [3] achieved a statistical tolerance analysis of straight bevel gear. Li Yinong [4] studied the influence of asymmetric mesh stiffness on dynamics of spiral bevel gear Transmission System.

In addition to these research works for the single stage model, there are studies on double or multistage model. Lian and Parker [5] modeled mesh stiffness in two stage spur gear systems. Walha [6] studied nonlinear dynamics behavior of two stage spur gear system. J. Kimotho [7] modeled a multi-stage spur gear Systems as a tool for design optimization. Driss [8] studied the effects of defects on helical two-stage gear system behavior.

Modeling of bevel gears is limited to one stage [3,4] although we found mechanisms of two stage bevel gears in processing machine such as the heads of milling, in special gearboxes and in transportation like helicopter. This work is devoted to a two-stage straight bevel gear which is modeled as faultless system in the first case and as a defected system in the second case in order to facilitate the diagnosis of this type of gear boxes.

In this research article, a two-stage straight bevel gear system was modeled. In the first, the dynamic behavior of a faultless system is presented. Then, we modeled the mesh stiffness depending on the angular position of contact's point and we calculated the tooth

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Notation

i	Gear indice
j	Block indice
k	Stage indice
F_{ji}	Below of the spherical involute on the base circle of bevel gear (i) of the block (j)
O_{ji}	Center of bevel gear (i) of the block (j)
Q_1	Crossing point of the two bevel gears (12) and (21)
Q_2	Crossing point of the two bevel gears (22) and (31)
S_1	Intersection of the arc of meshing with the circle head of the driving gear
S_2	Intersection of the arc of meshing with the circle head of the driving gear
I_1	Point slip-free rolling of pitch circles of the two bevel gears (12) and (21)
T_{e1}, T_{e2}	Gearing periods of the first stage and the second stage respectively
r_{12}	Radius of the sphere which contains two bevel gears (12) and (21)
r_{21}	Radius of the sphere which contains two bevel gears (12) and (21) ($r_{12} = r_{21}$)
r_{22}	Radius of the sphere which contains two bevel gears (22) and (31)
r_{31}	Radius of the sphere which contains two bevel gears (22) and (31) ($r_{22} = r_{31}$)
T_{ji}	Tangent point between plane of pressure and base circle of bevel gear (i) of the block (j)
Z_{ji}	Tooth number of bevel gear (ji)
α_1, α_2	Pressure angle of the first and the second bevel gears stages respectively
$\Phi_{12}S_1$	Angle between the planes (ts_1, O_{12}, S1) and (ts_1, O_{12}, O_{21})
$\Phi_{12}S_2$	Angle between the planes (ts_1, O_{12}, S2) and (ts_1, O_{12}, O_{21})
λ_{ji}	Angle $T_{ji}\,t\hat{s}_k\,S1$
χ_{ji}	Angle $T_{ji}\,t\hat{s}_k\,S2$
η_{ji}	Angle $O_{ji}\,t\hat{s}_k\,S2$
γ_{ji}	Angle $T_{ji}\,t\hat{s}_k\,I_k$
δ_{ji}	Half-angle of the pitch circle of bevel gear (i) of the block (j)
δ_{bji}	Half-angle of the base circle of bevel gear (i) of the block (j)
δ_{aeji}	Half-angle of the tip circle of bevel gear (i) of the block (j)
$\varepsilon_{\alpha 1}$, $\varepsilon_{\alpha 2}$	Contact ratio of the first and second bevel gears stages respectively
C_{aeij}	Effective outside diameter of bevel gear (i) of the block (j)

deflection. Then numerical results for the dynamic response are obtained by using Newmark algorithm. Finally, eccentricity defect, profile error and tooth crack are investigated on the dynamic behavior.

2. Model of two-stage straight bevel gear

Fig. 1 presents the lumped parameter model developed to study the dynamic behavior of the two-stage straight bevel gear system composed with three blocks. The first block ($j = 1$) is constituted of the drive wheel (11) which is connected to the bevel gear (12) via a shaft (1) which is supposed massless and torsional rigidity $k_{\theta 1}$. The bevel gear (21) is linked in the one hand to the bevel gear (12) via a teeth mesh stiffness $k_1(t)$ and it is linked to the bevel gear (22) via a shaft (2) which is massless and has stiffness of torsion $k_{\theta 2}$ in the other hand. So, the second block ($j = 2$) is constituted of two bevel gears (21) and (22) and the shaft (2). The bevel gear (31) is linked to the bevel gear (22) via a teeth mesh stiffness $k_2(t)$ and to the receiving wheel (32) via the massless shaft (3) and torsional rigidity $k_{\theta 3}$. So, the third block ($j = 3$) is constituted of the bevel gear (31), the receiving wheel (32) and the shaft (3). Each block (j) is supported by a flexible bearing with traction–compression stiffness k_{xj}, k_{yj} and k_{zj} and bending stiffness $k_{\psi j}$ and $k_{\Phi j}$.

3. Mesh stiffness modeling

This paragraph is devoted to explain the method for modeling the temporal fluctuations of the two meshing stiffness of straight bevel gears. Fig. 2 shows the arrangement of the studied two stage bevel gears, the crossing points Q_1 and Q_2 and meshing contact points C_1 and C_2 of each stage.

During the gears' meshing period, however, the number of pairs of meshing teeth alternates between one and two. Several authors, as in Ref. [1], represent mesh stiffness variation in spur gears as a periodic square wave function as in Fig. 4 where it is represented as the transition between single tooth pain in contact and double tooth pair in contact.

Furthermore, in this paper, the gear meshing is considered as a function of time and treated as a function of the angular position of the gear ($K_k(\phi_{ji}(t))$). In the rest of this paragraph, only the first meshing gear is described. If the contact point C_1 is between points S_1 and U_2, there are double tooth pair in meshing between U_1 and S_2. On the other hand, if C_1 is located between U_2 and U_1, there are single tooth pair in meshing (Fig. 3).

Fig. 4 describes first mesh stiffness fluctuation following $\phi_{12}(t)$ (rotational angle of gear (12)). Second mesh stiffness fluctuation can be expressed following $\phi_{22}(t)$ (gear (22) is the input gear on the second stage).