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Design of cams with negative radius follower using Bézier curves

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ABSTRACT

This paper discusses the application of Bézier curves for designing cams with a follower of negative radius $|R_f|$. This kind of cam-follower mechanisms is recommended in applications where the space is restricted and very high forces are involved. In particular, we propose a numerical method for optimizing the design of the cam profile using a Bézier ordinate as an optimization parameter. For the optimization problem addressed in this work, the follower lift is maximized avoiding the undercutting problem, but keeping the pressure angle lower than 41°. To that end, a parametric study has been performed using Bézier curves of orders $n = 6$ and $n = 8$, to ensure continuity for the second and third derivatives, respectively. It is found, for both orders $n = 6$ and $n = 8$, that the Bézier ordinate that maximizes the follower lift, for a given total rise angle β, is practically independent of the dimensionless prime circle radius $\alpha = R_a/|R_f|$.

1. Introduction

Cam-follower mechanisms are found in almost all mechanical devices and machines (i.e., agriculture, transportation equipment, textiles, packaging, machine tools, printing presses, automobile internal combustion engines, and more recently in micromachines such as microelectromechanical systems). Cams are widely used in many types of machines since they make it possible to obtain an unlimited variety of motions. Many different types of cam profiles are designed and manufactured depending on a machine's requirements. There are many ways to express a cam profile mathematically. The procedures to generate the cam profile are deeply explained in [1,2], where the influence of various parameters on undercutting and pressure angle problems is underlined.

The functions of standard motions to design cams include harmonic, cycloid, modified harmonic, trapezoidal, modified trapezoidal and polynomial [2]. Some of these curves are characterized by relatively smooth follower response at high speed. Others are plagued by vibrations and shocks even at a moderate speed, and the equations used to express these curves are usually rigid in their forms. In particular, some researchers generated the cam profile by periodic splines, and also a geometric relation between optimum parameters and displacement profile was found (see e.g. [4–8], among others). However, the Bézier curves may offer the best solution for the synthesis of cam-follower mechanisms. For example, [9–11] used Bézier curves to express the displacement, velocity and acceleration constraints that must be satisfied by the cam-follower set, whose motion characteristics and boundary continuities are to be decided by control points. Surprisingly, although the Bézier curves are powerful tool for designing cams, they are much less widespread in this engineering field than the modified trapezoidal curves or the spline functions.

On the other hand, not many studies are available about cam mechanisms with negative radius roller-follower. Carra et al. [3] developed a synthesis procedure, based on the modified trapezoidal curves, that allows finding a suitable combination of parameters (prime radius, roller radius, lift angle, etc.) in order to generate the correct motion law for these cam mechanisms without

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undercutting problems. Furthermore, they showed that such mechanisms are valuable in applications where, especially, the space is constrained and very high forces are involved.

Thus, to fill the gap in works on cam mechanisms with negative radius roller-follower, we propose an optimization procedure, based on the Bézier curves, to generate the correct motion law for this kind of cams without undercutting problems and by limiting the maximum pressure angle.

A procedure to obtain the cam profile, based on the curvature theory, is described in the next section. Section 3 is devoted to the mathematical foundations of the Bézier curves, while the results are given and discussed in Section 4. Finally, some conclusions are drawn in the last section.

2. Procedure to obtain the cam profile

A cam is a rotating or sliding piece in a mechanical linkage used especially in transforming rotary motion into linear motion or vice-versa. Fig. 1(a) shows the sketch of a classical cam with positive radius follower, while Fig. 1(b) shows the sketch of a cam with negative radius follower.

The cams with negative radius follower have certain advantages [3] over the classical cams: these mechanisms are valuable in applications where space is constrained; and very high forces are involved; the cam-follower mechanism does not present such severe restrictions with the maximum pressure angle, ϕ_{max}, which can reach 41°. Instead, in the case of classical cams the maximum pressure angle should be lower than 30°, to avoid jams into the guides and cause bending in the follower stem.

Next, let us describe briefly the procedure used in the current work to obtain the cam profile. Fig. 2 displays a cam-follower mechanism with negative radius follower $|R_f|$ (solid black/thin line). The reference system Oxy is assumed as fixed (attached to the frame of the mechanism) with its origin O placed at the cam axis of rotation, and on its axis Oy is placed the center C of the follower. However, the reference system Ox_1y_1 is attached to the cam. If we accept that the motion of the cam with respect to the frame is a rotation of angle θ, which angular velocity is $\boldsymbol{\omega} = \omega \boldsymbol{k}$, the motion of the reference system Oxy with respect to the system Ox_1y_1 is another rotation, which angular velocity is $\boldsymbol{\Omega} = -\omega \boldsymbol{k}$. The motion of the follower with respect to the reference system Oxy is a translation in the direction Oy. Hence, the position of the follower is fully defined whether the position of one of its points is known. Let us assume that the center of the follower is known (point C in Fig. 2), and its position vector relative to the reference system Oxy is,

$$\boldsymbol{r}_C = \begin{pmatrix} 0 \\ R_a - f(\theta) \end{pmatrix}, \tag{1}$$

where R_a is the radius of the prime circle (dashed black line in Fig. 2) and $f(\theta)$ is the motion law or function corresponding to the displacement or lift of the follower (see Section 3). Note that in Eq. (1) the motion law is subtracted from the prime circle radius in order to achieve the pitch curve. The geometrical locus of point C with respect to the reference system Ox_1y_1 is the pitch curve (dash-dot red line in Fig. 2). Furthermore, one can deduce that the position vector of the instantaneous center of the follower (point I in Fig. 2) with respect to the reference system Oxy is,

$$\boldsymbol{r}_I = \begin{pmatrix} -f_\theta(\theta) \\ 0 \end{pmatrix}. \tag{2}$$

The contact point between the follower (solid black/thin line in Fig. 2) and the cam (which profile is the solid blue/thick line in Fig. 2), is obtained taking into account that the cam profile is the envelope of the circumferences that represent the follower during

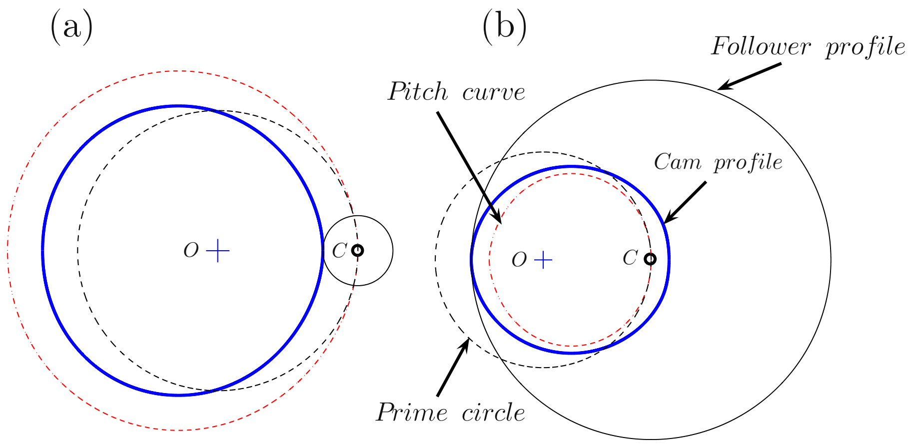

Fig. 1. Types of cam-follower mechanisms: (a) cam with a positive radius follower; (b) cam with a negative radius follower. The curves correspond to: the cam profile (solid blue/thick line); the follower profile (solid black/thin line); the pitch curve (dash-dot red line); the prime circle (dashed black line). The center of rotation of the cam (O) and the follower (C) is also shown.