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Free vibration analysis of a nonlinear slender rotating shaft with simply support conditions

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ABSTRACT

The free vibration analysis of a nonlinear slender rotating shaft with simply support conditions is studied. In the slender rotating shaft the effect of shear deformation is negligible. Accordingly, in the modeling of the system rotary inertia and gyroscopic effect are considered. The equations of motion are derived by the extended Hamilton principle. The nonlinear system analyzed utilizing multiple scales method. The forward and backward nonlinear frequencies of the slender rotating shaft are obtained. It is seen that for natural vibration of a slender rotating shaft, backward and forward modes are involved.

1. Introduction

The vast majority of industrial devices have a rotating shaft. Also, the rotating part of the system is one of the important sources of vibrations. Furthermore, in order to analyze a rotating system, it is necessary to first gain the natural frequencies of the system. Accordingly, natural vibration analysis for the nonlinear rotating system is mandatory.

In [1] the authors investigated the free and forced response of a viscoelastic spinning Rayleigh shaft. The closed polynomial form of frequency equation and integral expressions subjected to a general forcing function were derived. Karunendiran and co-worker [2] analyzed the free vibrations of shafts on resilient bearings. Using the Timoshenko beam theory the exact frequency equation in the complex compact form was derived.

In Ref. [3,4] the free vibrations and stability of an internally damped rotating shaft with general boundary conditions were investigated. Also, in Ref. [5] the critical speeds and mode shapes of a spinning Rayleigh beam for six general boundary conditions were analytically investigated.

In [6–8] the free vibrations of a rotating beam with random properties, the vibration and reliability of a rotating beam with random properties under random excitation, and the free vibrations of an in-extensional rotating shaft with nonlinear curvature and inertia were considered.

Ji and Zu [9] investigated the free and forced vibrations of a nonlinear rotor-bearing system by using the method of multiple scales. The free and force vibration analysis of a rotating disk-shaft system with linear elastic bearings was investigated by Shabaneh and Zu [10]. The shaft was modeled by the Timoshenko beam theory and visco-elastic supports were represented by Kelvin–Voigt model. Chang-Jian et al. [11] considered the vibrations of a rotor bearing system with nonlinear suspension. To analyze the chaotic dynamics, the Poincare map, Liapunov exponents and bifurcation diagrams were used. It was shown that the stability of the system varies with dimensionless speed ratio parameter. In addition, the quasi-periodic, chaotic motions, and periodic motions were obtained. Al-

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Shudeifat et al. [12] considered vibrations of a rotor-bearing-disk system with crack. The effect of crack depth on the vibrations' amplitude and whirling orbit shapes was investigated. It was realized that large vibrations' amplitude at the critical values of crack depth and rotor speed occurs in a slightly imbalance system. A non-linear spring and linear damping to model the non-linear bearing pedestal were used. Shaw et al. [13] analyzed the stability and bifurcations of a balanced nonlinear rotating shaft made of a viscoelastic material. To examine the post-critical behaviors, the center manifold theory was applied to the nonlinear equations of motion. Numerical simulation was carried out for static and free vibration cases.

In [14,15], Kurnik investigated self-excited vibrations of a rotating non-linear shaft by bifurcations theory. In these works the internal friction of shaft and material nonlinearities were considered.

Grybos [16] considered the effect of the shear deformation and rotary inertia of a rotor on its critical speeds. Jei and Leh [17] investigated the whirl speeds and mode shapes of a uniform asymmetrical Rayleigh shaft with asymmetrical rigid disks and isotropic bearings. Kim et al. [18] studied the free vibration of a rotating tapered composite Timoshenko shaft. Park et al. [19] investigated the linear vibration of the wind-turbine blade. Multi-body dynamics method was used to derive the equations of motion. To verify the accuracy of this method, numerical problems were solved. Lee et al. [20] analyzed the aerodynamic characteristics of a counter-rotating turbine. Three kinds of rotor configurations, which are 2-bladed single, 4-bladed single and counter-rotating rotor, were compared using numerical methods. The aerodynamic feasibility of a counter-rotating wind turbine was considered using numerical calculations of induction factors and power coefficients for each rotor.

According to the literatures, it seems that investigation of the free vibrations of a shaft with stretching nonlinearity would give out many interesting results. In this investigation, free vibrations of a slender rotating shaft with stretching nonlinearity are studied. In the slender rotating shaft the effect of shear deformation is negligible. Accordingly, in the modeling of the system rotary inertia and gyroscopic effect are considered. The equations of the motion are derived using the extended Hamilton principle. To simplify the analysis of the system the equations are transformed to the complex plane. The nonlinearity is due to extensionality of the slender rotating shaft. To analyze the free vibrations, the method of multiple scales is applied to the partial differential equations of the motion. The forward and backward nonlinear frequencies of the slender rotating shaft are obtained. It is seen that for natural vibration of a slender rotating shaft, backward and forward modes are involved. In addition, the effect of rotary inertia on the response of the system is investigated.

Nomenclatures

A	cross section area
A_{11}	longitudinal stiffness
c	external damping coefficient
D_{11} , D_{22}	torsional and flexural stiffness
e	strain along the neutral of the shaft
E	elasticity modulus
G	shear modulus
E	the second moment of area
I_1 , I_2	polar and diametrical mass moment of inertia
k_i $(i = 1..3)$	shaft curvatures
l	length of rotating shaft
m	mass per unit length of shaft
u	longitudinal displacement
v, w	transverse displacements
ψ, θ, β	Euler angles
ρ	mass density
Ω	spinning speed

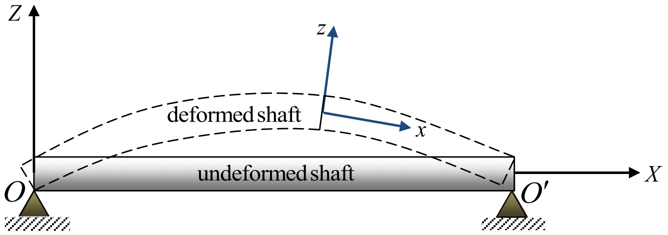

Fig. 1. Schematic of a rotating shaft with local and global coordinates.