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The significance of the configuration space Lie group for the constraint satisfaction in numerical time integration of multibody systems

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ABSTRACT

The dynamics simulation of multibody systems (MBS) using spatial velocities (non-holonomic velocities) requires time integration of the dynamics equations together with the kinematic reconstruction equations (relating time derivatives of configuration variables to rigid body velocities). The latter are specific to the geometry of the rigid body motion underlying a particular formulation, and thus to the used configuration space (c-space). The proper c-space of a rigid body is the Lie group $SE(3)$, and the geometry is that of the screw motions. The rigid bodies within a MBS are further subjected to geometric constraints, often due to lower kinematic pairs that define $SE(3)$ subgroups. Traditionally, however, in MBS dynamics the translations and rotations are parameterized independently, which implies the use of the direct product group $SO(3)\times\mathbb{R}^3$ as rigid body c-space, although this does not account for rigid body motions. Hence, its appropriateness was recently put into perspective.

In this paper the significance of the c-space for the constraint satisfaction in numerical time stepping schemes is analyzed for holonomically constrained MBS modeled with the 'absolute coordinate' approach, i.e. using the Newton–Euler equations for the individual bodies subjected to geometric constraints. The numerical problem is considered from the kinematic perspective. It is shown that the geometric constraints a body is subjected to are exactly satisfied if they constrain the motion to a subgroup of its c-space. Since only the $SE(3)$ subgroups have a practical significance it is regarded as the appropriate c-space for the constrained rigid body. Consequently the constraints imposed by lower pair joints are exactly satisfied if the joint connects a body to the ground. For a general MBS, where the motions are not constrained to a subgroup, the $SE(3)$ and $SO(3)\times\mathbb{R}^3$ yield the same order of accuracy. Hence an appropriate configuration update can be selected for each individual body of a particular MBS, which gives rise to tailored update schemes. Several numerical examples are reported illustrating this statement.

The practical consequence of using $SE(3)$ is the use of screw coordinates as generalized coordinates. To account for the inevitable singularities of 3-parametric descriptions of rotations, the kinematic reconstruction is additionally formulated in terms of (dependent) dual quaternions as well as a coordinate-free ODE on the c-space Lie group. The latter can be solved numerically with Lie group integrators like the Munthe-Kaas integration method, which is recalled in this paper.

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1. Introduction

The seemingly simple problem addressed in this paper is how to numerically reconstruct the finite motion of a constrained rigid body within a MBS from its velocity field so that the overall system of geometric constraints is satisfied. When a rigid body moves it performs a translation together with a rotation since a general rigid body motion is a screw motion, with coupled rotation and translation. Even though standard numerical integration schemes for MBS neglect the geometry of Euclidean motion in the sense that, within the integration schemes, the position and orientation updates are performed independently. Whether or not their dependence is respected has to do with the geometric model used to represent rigid body motions, i.e. with the configuration space (c-space) Lie group. It is known that rigid body motions form the Lie group $SE(3)$ [50,59].

With the 'absolute coordinate' formalism (i.e. representing the spatial configuration of each body by a set of six generalized coordinates) the equations governing the dynamics of a constrained MBS comprising n rigid bodies are commonly written in the form

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{V}}+\mathbf{J}^T\boldsymbol{\lambda}=\mathbf{Q}(\mathbf{q},\mathbf{V},t) \tag{1a}$$

$$\mathbf{V}=\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} \tag{1b}$$

$$h(\mathbf{q})=0. \tag{1c}$$

The $N=6n$ dimensional coordinates vector $\mathbf{q}=(\boldsymbol{\theta}_i,\mathbf{r}_i)\in\mathbb{V}^N$ comprises the position vector $\mathbf{r}_i$ and the vector $\boldsymbol{\theta}_i$ consisting of 3 (or 4 dependent) rotation parameters for body $i=1,\ldots,n$, and $\mathbf{V}=(\boldsymbol{\omega}_i,\mathbf{v}_i)\in\mathbb{R}^N$ is composed of the angular and linear velocity vectors ω_i and $\mathbf{v}_i$, respectively. The matrix $\mathbf{J}$ is the constraint Jacobian corresponding to the system (1c) of geometric constraints.

Eqs. (1) constitute a DAE system on the coordinate manifold $\mathbb{V}^N$ considered as vector space. From a kinematic point of view this formulation raises two issues regarding their numerical solution:

1. The motion of the MBS is deduced from the velocity $\mathbf{V}$ by the *kinematic reconstruction equations* (1b). The accuracy of their numerical solution depends directly on the underlying geometry of rigid body motions, which is encoded in the mapping $\mathbf{A}$. In the standard MBS formulation the rotations and positions are reconstructed separately according to

$$\begin{pmatrix}\boldsymbol{\omega}_i\\ \mathbf{v}_i^s\end{pmatrix}=\begin{pmatrix}\mathbf{B}_i\ (\boldsymbol{\theta}_i) & 0\\ 0 & \mathbf{I}\end{pmatrix}\begin{pmatrix}\dot{\boldsymbol{\theta}}_i\\ \dot{\mathbf{r}}_i\end{pmatrix}, i=1,\ldots,n. \tag{2}$$

   The underlying geometry is that of $SO(3)\times\mathbb{R}^3$, which does not account for the coupling of rotations and translation inherent to screw motions. Nevertheless, the kinematic (2) correspond to a valid parameterization of rigid body *configurations*. The interdependence of ω_i and $\mathbf{v}_i^s$ is ensured by solving (1a) and (1c), and an analytic solution of (2) correctly reflects the bodies' screw motions. However, when (1) are solved numerically with a finite step size, and (1b) is used to predict finite (screw) *motion* increments, also the kinematic reconstruction equations (1b) must properly reflect the geometry of screw motions. Moreover, (2) can only predict the finite motion if $\mathbf{r}_i$ are the coordinates of a point on the rotation axis, as for instance in the case of an unconstrained body with its body-fixed reference frame located at the COM. A generic motion of a constrained body, as part of a MBS, will not comply with the decoupling assumption encoded in (2). The matrix $\mathbf{B}$ is specific to the rotation parameterization. If Euler angles are used, for instance, $\mathbf{B}$ corresponds to the kinematic Euler equations [42]. Consequently, the kinematic reconstruction equations (1b) shall be amended in order to respect the interrelation of rotationsand translations, which boils down to the appropriate choice of the rigid body configuration space being a Lie group. The implications of using the Lie group $SE(3)$ as well as $SO(3)\times\mathbb{R}^3$ are studied in this paper.
2. The second issue regards the violations of the constraints (1c) that occur when numerically solving (1). This has been a central problem in numerical MBS dynamics. However, the investigations have exclusively been focused on reducing or correcting constraint violations by means of stabilization and projection methods [2–5,15,63] rather than aiming to avoid such violations. It is immediately clear that the constraint satisfaction is affected by the accuracy with which the finite motions are reconstructed from the velocity field $\mathbf{V}$ solving Eq. (1a), which indeed depends on the feasibility of the relation (1b). Even more, besides the accuracy with which the system dynamics is captured by the numerical integrator, it is crucial to ensure the kinematic consistency of the MBS, thus the constraint satisfaction is imperative. This is the focus of this paper. In this respect it is important to observe that the majority of mechanisms is built with lower kinematic pairs (Reuleaux pairs). The latter are characterized by their isotropy groups, i.e. subgroups of $SE(3)$ leaving the contact surface invariant. It is clear that, if a numerical update step does not respect these motion groups, the lower pair constraints will be violated.

The reconstruction equations (1b) represent a first-order relation, and from a computational perspective the question arises whether the decoupling significantly affects the accuracy of the numerical solution of (1). The goal of this paper is to study the extent to which different forms of this first-order relation affect the reconstruction of finite motions of a constrained MBS with numerical