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Reliability design and sensitivity analysis of cylindrical worm pairs

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ABSTRACT

The reliability and the reliability sensitivity of a cylindrical worm drive are considered in the present paper. To estimate them, some new mathematical methods are proposed. Depending on these approaches, a practical and simple technique to perform the estimation is taken shape. The application of such technique does not require knowing the probability distributions of the basic design parameters. The evaluation can be implemented so long as the mean values and the variances of the basic design arguments are known. In the bargain, the situation that the standardized random variable obeys an arbitrary probability distribution can be handled by taking advantage of the technique suggested as well. The numerical results show that the traditional excogitation methodology can in general give a favorable outcome with higher reliability. Generally speaking, as for an enclosed cylindrical worm gearing, the contact fatigue reliability is usually, more or less, higher than the bending fatigue reliability. In addition increasing the center distance and the module is beneficial to improve the reliability. Meanwhile the greater variance of the design parameters may lead to lower reliability as to a worm gear pair.

1. Introduction

The cylindrical worm drive, as shown in Fig. 1, is an important and fundamental machine element, which is widely used in almost every industrial sector. The failure of worm pairs often leads to a poor performance of the related parent systems, and sometimes even incurs serious accidents and subsequently great economic losses. Therefore, the reliability design of a worm gearing should have important implications but the relevant study has rarely been reported in the mainstream journals for a long time. In principle, it is not easy to calculate the reliability of a worm pair accurately. One of the causes is that the probability distribution of the strength for the worm gear material, for example tin bronze, is often unknown due to the lack of the relevant observed data.

In pace with the advancement of the reliability theory, the situation began to change to some degree. In the nineties of the last century, Liu [1] proposed a practical method to estimate the lower limiting value of the reliability for a worm gear set by means of the variation coefficient and the mean value of the safety coefficient.

In civil engineering, the method to evaluate the reliability of structures based on the so-called reliability index has driven to maturity [2–4]. Borrowing this strategy, Zhou et al. [5] estimated the reliability for a cylindrical worm drive in the near past.

In the present study, the problem of assessing the reliability and analyzing the reliability sensitivity of a cylindrical worm drive is taken into account. To solve this problem, some new methods in the mechanical reliability theory are put forth, for instance, the method to compute the higher order moments of the state function based on its Taylor expansion in the case that the design parameters are mutually independent, the novel technique to evaluate the reliability by employing the Edgeworth series in the case of arbitrary probability distribution, and the approach to calculate the reliability sensitivity with the help of the derivative of a scalar with respect to a

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Fig. 1. A cylindrical worm pair.

vector or a matrix. Then by using the new methods proposed a detailed numerical simulation research is conducted on the reliability and the reliability sensitivity of a cylindrical worm pair.

2. Theoretical foundation to compute reliability

By definition, the problem of the reliability of a worm drive can be boiled down to the calculation of a multi-fold integral as follows

$$R=\int\limits_{g\left(\vec{\boldsymbol{X}}\right)>0} f\left(\vec{\boldsymbol{X}}\right)\mathrm{d}\vec{\boldsymbol{X}}, \tag{1}$$

where R is the reliability. Concurrently $\vec{\boldsymbol{X}}=[X_1,X_2,\cdots,X_n]^T$ is a random column vector, whose components are the basic design variables. $f\left(\vec{\boldsymbol{X}}\right)$ is the joint probability density function of $\vec{\boldsymbol{X}}$. $g\left(\vec{\boldsymbol{X}}\right)$ is a multivariate function and denotes the state function. In some literature, $g\left(\vec{\boldsymbol{X}}\right)$ is also called the performance function [2].

Letting $\vec{\boldsymbol{\mu}}$ be the mean value column vector of the random column vector $\vec{\boldsymbol{X}}$ yields

$$E\vec{\boldsymbol{X}}=[EX_1,EX_2,\cdots,EX_n]^T=[\mu_1,\mu_2,\cdots\mu_n]^T=\vec{\boldsymbol{\mu}}, \tag{2}$$

where n is the number of the design parameters.

In this study, the basic design variables for a worm drive are assumed to be mutually of independence. Therefore the covariance matrix of random vector $\vec{\boldsymbol{X}}$ degrades into a diagonal matrix in the form

$$Cov\left(\vec{\boldsymbol{X}}\right)=diag\left[\sigma_1^2,\sigma_2^2,\cdots\sigma_n^2\right], \tag{3}$$

where σ_j^2 is the variance of the random variable X_j ($j=1,2,\cdots,n$), and $DX_j=\sigma_j^2$.

If the experimental data is not sufficient or available, the standard variance of the stochastic design arguments may be estimated by virtue of the variation coefficient, C, after the mean value of the corresponding stochastic parameter has been determined. The primary principles are listed as follows [1,6,7].

(a) For a mechanical property parameter, in general, $C=0.05$.
(b) The standard deviation of a geometry parameter can be ascertained by using the tolerance standard. If no tolerance standard, in general, $C=0.015$.
(c) For other parameters, in general, $C=0.033$.

In accordance with the definition of the state function, $g\left(\vec{\boldsymbol{X}}\right)$ represents the safe state or the failure state of a worm pair, i.e.

$$\left.\begin{array}{l} Z=g\left(\vec{\boldsymbol{X}}\right)\le 0 \quad \text{failure state} \\ Z=g\left(\vec{\boldsymbol{X}}\right)>0 \quad \text{safe state} \end{array}\right\},$$

where Z is a scalar random vector.