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Short communication

Mobility analysis of generalized angulated scissor-like elements with the reciprocal screw theory

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ABSTRACT

This paper deals with the planar closed loop linkage consisting of a series of scissor-like elements connected by revolute joints. Because every generalized angulated element (GAE) subtends a constant angle during the motion, every angulated link was assumed as a PRRP linkage which has two prismatic joints and two revolute joints. Therefore, the two PRRP linkages of the GAE are individually movable with a single degree of freedom. The mobility of two types of GAEs was investigated with the method based on the screw theory. It has been proven that both types of GAEs are movable because the terminal constraints exerted to the common joint by the two linkages are equal.

1. Introduction

Foldable structures, which can be transformed from a closed configuration to an expanded form, are developed from the earliest modern stages with Pinero's movable theater in 1961 [1]. Especially during the last decade, the retractable roof structures are widely used in long-span space structures [2]. On the other hand, with the further development of aerospace science and technology, deployable structures are of even greater interest in aerospace industries [3,4].

Generally, the deployable structure based on the scissor-like element is the most commonly used form in engineering [5–8]. A scissor-like element in two dimensional forms is shown in Fig. 1(a). The axis of the revolute joint is perpendicular to the plane of the structure. The element has one degree-of-freedom and can be folded and deployed freely. Besides the foldable truss and dome structures [9], Zhao et al. [10,11] proposed a foldable stair, which consists of a number of identical deployable scissor-like elements. In the early 1990's, Hoberman [12,13] invented and patented a method for constructing loop assemblies formed by the modified scissor-like element, which consists of a pair of identical angulated rods connected by a revolute joint as shown in Fig. 1(b). Therefore, the unit is also called angulated scissor element or Hoberman's unit. You and Pellegrino [14] noted that the unit subtends a constant angle as their rods rotate while maintaining the end pivots on parallel lines. Thus the Hoberman's unit can create a closed-loop mechanism, which is called Hoberman's Linkage as shown in Fig. 2. They also proposed two types of generalized angulated elements (GAEs), which subtend a constant angle during folding, but afford much greater freedom of shapes than Hoberman's unit. The triangles of the two angulated rods of Type I GAE are isosceles triangles, i.e. AE = DE and BE = CE, as shown in Fig. 3. The triangles of the two angulated rods of Type II GAE are similar triangles, i.e. AE/BE = DE/CE and ∠AEB = ∠DEC. As shown in Fig. 4, the lengths of BE and

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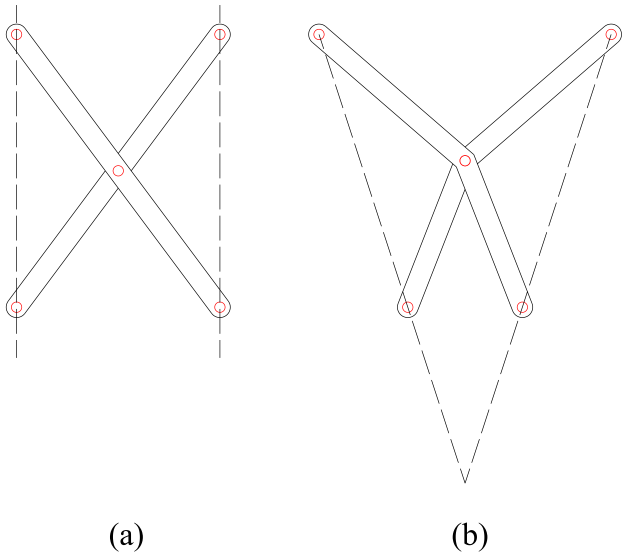

Fig. 1. Scissor-like elements.

AE are p and q. For Type II GAE, AE/BE = DE/CE = w. Then the lengths of CE and DE are wp and wq. The planar closed loop linkages which include Type I GAEs and Type II GAEs are given in Fig. 5.

The focus of this paper is on the mobility analysis of this planar closed loop linkage. It is pertinent to point out that the kinematic treatment of linkages is receiving increasing attention in recent years [15–18]. Langbecker [5] proposed the foldability equation of deployable scissor structures, which is formulated using a purely geometric approach. Patel and Ananthasuresh [16] presented a kinematic theory behind Hoberman's and other inventions related to planar, radially foldable linkages. They showed that known types of foldable linkages can be derived by using a simple algebraic equation and even simpler design criteria. Based on the corresponding mobility conditions, Mao et al. [17] produced mobile double chains with both even and odd numbers of intersecting scissor-like pairs. A numerical algorithm was used by Nagaraj et al. [18] to evaluate the degree-of-freedom of pantograph masts by obtaining the null space of a constraint Jacobian matrix. Moreover, the principles of screw theory can also be used to study the degree-of-freedom of mechanism. Dai et al. [19–21] studied the mobility of a general type of foldable mechanisms using the screw theory. Zhao et al. [22] applied the screw theory to study the degree-of-freedom of simple planar linkage and the mechanism theory of forming the spatial deployable units utilized in flat, cylindrical and spherical deployable structures. But they only focused on the scissor-like elements with straight rods. Then Cai et al. [23] extend this theory to the mobility analysis of the Hoberman's angulated unit. Dai et al. [24] and Wei et al. [25,26] also used the reciprocal screws to study the mobility of the Hoberman structure and its variant. However, the scissor-like elements studied in the previous literatures are very simple and the geometry of these elements is symmetry. Two types of GAEs proposed by You and Pellegrino [14], which are not considered in these literatures, have asymmetry geometry and complex kinematic behavior. The mobility of GAEs will be studied in this paper with the screw theory. Prior to probing

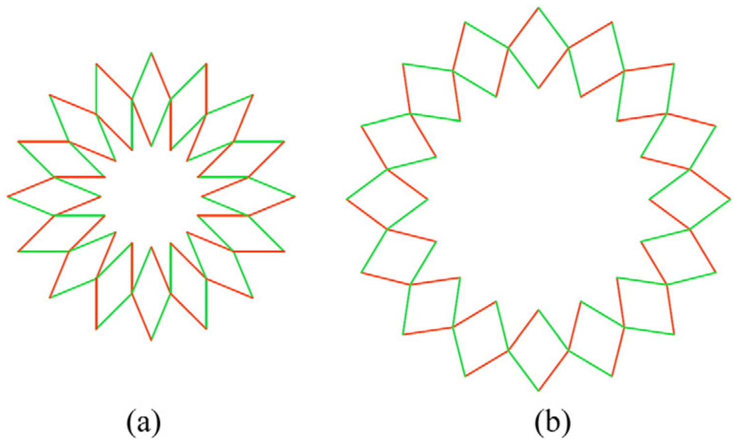

Fig. 2. Hoberman's Linkage.