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Optical geometry approach for elliptical Fresnel lens design and chromatic aberration

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ABSTRACT

This research formulates an elliptical-based Fresnel lens concentrator system using optical geometry and ray tracing technique. The author incorporates solar spectrum with the refractive indices of lens materials to form different color mixes on the target plane. The model illustrates the solar spectrum distributions under the Fresnel lens. It can be used to investigate each spectral segment's distribution patterns and helps to match the concentration patterns of different wavelengths to different solar energy applications.

1. Introduction

The Fresnel lens incorporates a number of facets shaped as prisms, each of which is designed to refract the incoming light to a small focal area. Concentration occurs as all facets on the lens direct their light beams to the same focal area. The imaging accuracy of such non-imaging lenses is of little concern as long as the lens captures radiation within the target. The Fresnel lens can take many forms, and has proved to be a useful optical device with many applications. Today, the Fresnel lenses are widely used as solar concentrators for their relatively high optical efficiency, light weight, and low cost. Photovoltaic applications that call for optical concentrators are often equipped with such lenses.

In a flat lens, the first face incident angle is always zero and refraction does not occur there. The entire refraction burden for a flat Fresnel lens is shifted to the second face. When the incident angle of the second face approaches $\sin^{-1}(1/\tilde{n})$, the beam spread becomes very large and a part of the flux misses the designated target area.

Cosby [1] pointed out that the required target width of a Fresnel lens concentrator generally decreases and peak concentration increases with increasing lens curvature. In order to vary the curvature of the lenses with respect to their focal plane, Cosby used circular sections with radii at points above or below the focal plane.

James and Williams [2] use both optical sensor analysis and calculations to study the concentrated radiation from flat and curved base point-focus Fresnel lenses. The concentration profile of their lens is measured with a solar cell behind a pinhole in the target plane. They conclude that as more refraction takes place on the first surface of the lens and less takes place on the second surface, both total internal reflection and chromatic aberration decrease and thus the system would allow the use of a smaller cell. Some researchers like Luque and Lorenzo [3] have found that lenses with elliptical shapes have some interesting properties. Eriksen [4] has shown that one of them almost exactly duplicates the curvature required to achieve maximum transmission. Leutz and Suzuki [5] address the function and design of dome Fresnel lenses in some detail. They also assume that the ideal lens will have an elliptical shape.

2. Ray tracing and elliptical-based lens

In this paper, the author extends the work of Yeh [6] and integrates ray tracing technique to formulate a complete elliptical-based Fresnel lens solar concentrating system with the intention to identify the curvature of maximum light transmission for lens performance evaluation.

Apply Snell's law to Fig. 1 and obtain

$$\sin\varphi_i = \tilde{n}\sin\varphi_r \quad \text{(for face AB)} \tag{1}$$

$$\tilde{n}\sin\varphi_i' = \sin\varphi_r' \quad \text{(for face AC)} \tag{2}$$

Where φ_i and φ_i' are the incident angles on the faces AB and AC, respectively; φ_r and φ_r' are the refraction angles on the faces AB and AC, respectively; and $\tilde{n}$ is the refractive index of specific wavelength interval when passing the lens.

Depending on the parameter chosen, certain segment of the solar spectrum can be focused in the center of the focal area. All other parts of the spectrum will be more or less off focus. The wavelength of the spectrum segment, that is correctly focused, is

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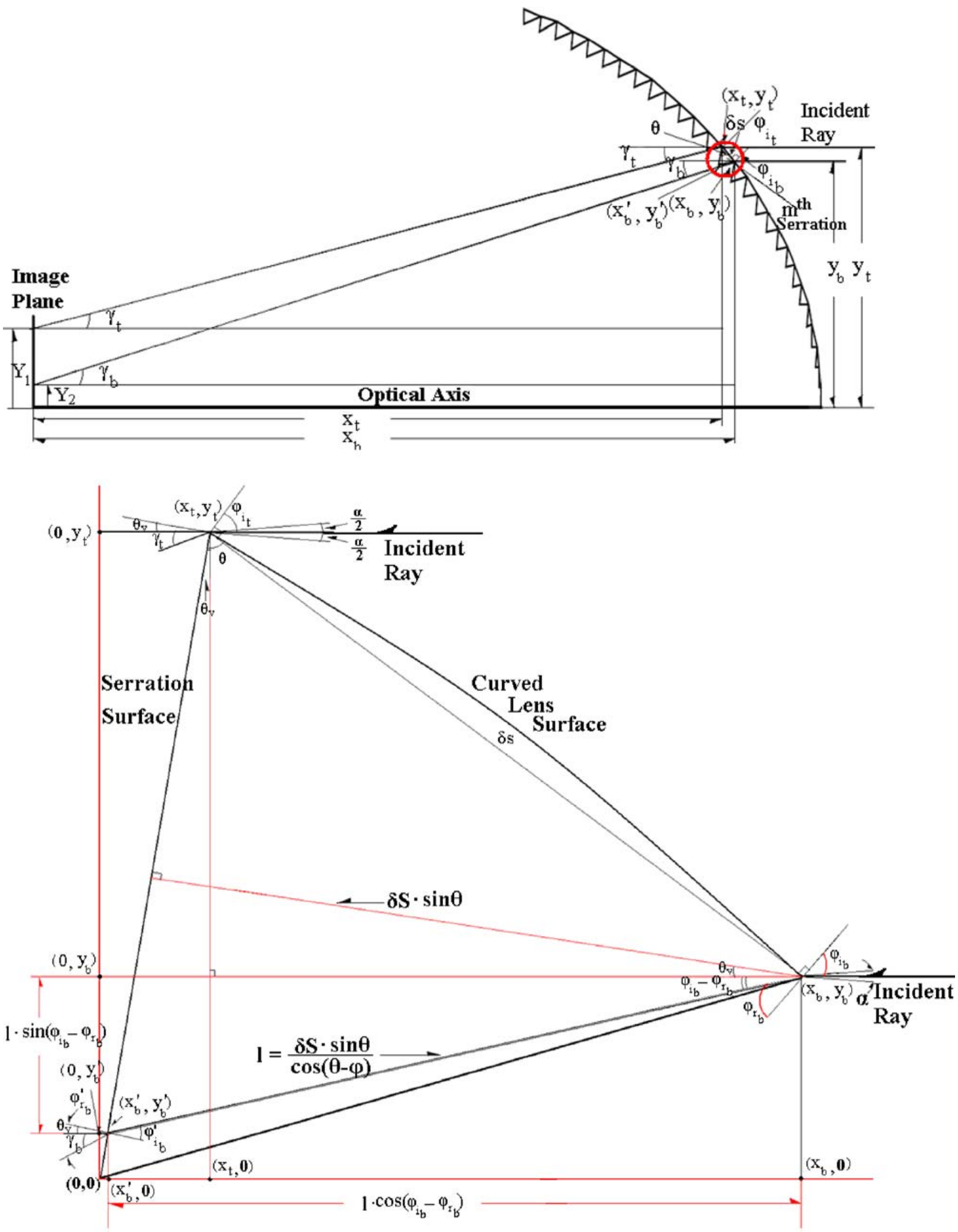

Fig. 1. Schematic of the extreme ray tracing through a facet on an elliptical-based curve lens (top) with the diagram of a facet that depicts the geometry of the extreme ray tracks through the prism (bottom, enlarged from the circled area of the top). Note that the illustration only depicts the case of $Y_1>0$ and $Y_2>0$, where both Y_1 and Y_2 fall on the same side of the optical axis. Such case of under refraction applies to the wavelengths that are longer than the design wavelength. There are also other cases where $Y_1>0$ and $Y_2<0$ (applies to the wavelengths near the design wavelength) or $Y_1<0$ and $Y_2<0$ (over refraction, applies to the wavelengths that are shorter than the design wavelength) applies.

referred to as design wavelength. The refractive index of the lens that focuses the design wavelength to the focal area center is called the design index. These two design parameters are mutually related and will be used interchangeably later on in this work.

The following relationships exist as per Fig. 1:

$$\varphi_i' = \varphi_i - \varphi_r + \theta_v \tag{3}$$

$$\varphi_r' = \gamma + \theta_v \tag{4}$$

$$\theta = \varphi_i + \theta_v \tag{5}$$

where γ is the turning angle of the light; θ_v is the complement of the facet angle with respect to the optical axis; and θ is the facet angle (i.e. the angle between the faces AB and AC).

The goal of the Fresnel lens design is to derive the facet angle of every facet on lens. This task can be accomplished by first deriving θ_v.

Substitute Eqs. (3) and (4) into Eq. (2) to obtain:

$$\tilde{n}\sin(\varphi_i - \varphi_r + \theta_v) = \sin(\gamma + \theta_v) \tag{6}$$

Eq. (6) leads to

$$\tilde{n}\sin(\varphi_i - \varphi_r)\cos\theta_v + \tilde{n}\cos(\varphi_i - \varphi_r)\sin\theta_v = \sin\gamma\cos\theta_v + \cos\gamma\sin\theta_v \tag{7}$$

Divide both sides of Eq. (7) with $\cos\theta_v$ and reach:

$$\tilde{n}\sin(\varphi_i - \varphi_r) + \tilde{n}\cos(\varphi_i - \varphi_r)\tan\theta_v = \sin\gamma + \cos\gamma\tan\theta_v \tag{8}$$