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Spectral fatigue life estimation for non-proportional multiaxial random loading

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A B S T R A C T

A frequency-domain High-Cycle Fatigue (HCF) criterion based on the critical plane approach is here proposed to estimate fatigue life of smooth metallic structural components under multiaxial random loading. The procedure consists of the following three steps: (a) definition of the critical plane; (b) PSD evaluation of an equivalent normal stress; and (c) computation of the fatigue life. A new formulation to define the critical plane is adopted in order to improve the lifetime estimation. The criterion is validated through comparison of the obtained numerical results with experimental fatigue data available in the literature for structural steel specimens subjected to a combination of random non-proportional bending and torsion.

1. Introduction

The type of loading experienced by many engineering structural components during their service life may be random and non-proportional. This is the case for metallic structures such as pressure vessels, nuclear and pressure water reactors, gas turbines, and automobile crankshafts [1,2]. Fatigue analysis under multiaxial random loading is a research topic still open [3–10]: as a matter of fact, the multiaxial state of stress under random loading requires the use of cycle counting techniques and models which are significantly more complex than those used to estimate life under uniaxial loading.

Multiaxial fatigue criteria have historically been formulated according to the time-domain approach [11–14]. Time-domain procedures are based on cycle counting methods and damage accumulation rules and, furthermore, require the knowledge of the time histories of the local stress tensor components. Note that many records are needed in order to obtain reliable statistical parameters of the loading process.

On the other hand, multiaxial fatigue criteria formulated according to the frequency-domain approach (often named spectral methods) [3,9,15–21] are alternative tools to treat loading of random type [22]. Frequency-domain procedures require no cycle counting methods but they need damage accumulation rules and, starting from the Power Spectral Density (PSD) function of local stress tensor components, an estimation of damage is directly obtained. Such features make the above criteria much more computationally efficient than the time domain ones [15], still providing high levels of accuracy.

Hybrid frequency-time domain methods for predicting multiaxial fatigue life are also available in the literature [4].

Some multiaxial fatigue criteria originally developed in time domain have been reformulated in frequency domain as multiaxial spectral methods [6], and may be based on either an equivalent uniaxial stress [15–21] or stress invariants [23,24].

Spectral fatigue life estimation for non-proportional multiaxial random loadings is the subject of the present paper. For such a purpose, the authors here propose a fatigue criterion based on an equivalent uniaxial stress evaluated on the critical plane [25–27]. This criterion is a frequency-based reformulation of the original time-domain method presented in Refs. [28–30].

The procedure consists of the following three steps:

(a) definition of the critical plane;
(b) PSD evaluation of an equivalent normal stress on the critical plane;
(c) computation of the fatigue life.

The orientation of the critical plane is connected to both averaged principal stress directions related to critical points of the structural component being examined and the fatigue properties of material. The latter dependence is taken into account through a rotational angle, δ, whose expression depends on material fatigue limits under fully reversed normal stress ($\sigma_{af,-1}$) and shear stress ($\tau_{af,-1}$) [28].

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Nomenclature

$\mathbf{C} = \mathbf{C}(\phi, \theta, \psi)$	rotation matrix from *PXYZ* to *PX′Y′Z′*
$\tilde{\mathbf{C}} = \tilde{\mathbf{C}}(\phi, \theta, \psi)$	rotation matrix from $P\hat{1}\hat{2}\hat{3}$ to *Puvw*
$E[D]$	expected fatigue damage per unit time
$p_a(s)$	marginal amplitude distribution of the counted equivalent stress cycles
PXYZ	fixed reference system
PX′Y′Z′	rotated reference system
$P\hat{1}\hat{2}\hat{3}$	reference system of the weighted mean principal stress axes
Puvw	reference system attached to the critical plane
$R_{i,j}(\tau)$	auto/crosscorrelation functions
$S_{eq}(\omega)$	equivalent PSD function
$\mathbf{s}_{xyz}(t)$	stress vector referred to the reference system *PXYZ*
$\mathbf{s}_{x'y'z'}(t)$	stress vector referred to the reference system *PX′Y′Z′*
$\mathbf{s}_{uvw}(t)$	stress vector referred to the reference system *Puvw*
$\mathbf{S}_{xyz}(\omega)$	Power Spectral Density (PSD) matrix of $\mathbf{s}_{xyz}(t)$
$S_{i,j}(\omega)$	coefficients of the $\mathbf{S}_{xyz}(\omega)$ matrix
$\mathbf{S}_{x'y'z'}(\omega)$	Power Spectral Density (PSD) matrix of $\mathbf{s}_{x'y'z'}(t)$
$S_{i',j'}(\omega)$	coefficients of the $\mathbf{S}_{x'y'z'}(\omega)$ matrix
$S_{3',3'}$	PSD function of the normal stress $\sigma_{z'}$
$S_{6',6'}$	PSD function of the shear stress $\tau_{y'z'}$
$\mathbf{S}_{uvw}(\omega)$	Power Spectral Density (PSD) matrix of $\mathbf{s}_{uvw}(t)$
$S_{3'',3''}$	PSD function of the normal stress σ_w
$S_{6'',6''}$	PSD function of the shear stress τ_{vw}
t	time
T	observation time interval
T_{cal}	calculated fatigue life
T_{exp}	experimental fatigue life
γ	rotation about $\mathbf{w}$-axis
δ	angle between the averaged direction $\hat{1}$ and the normal $\mathbf{w}$ to the critical plane (Fig. 2b)
$\delta_{L,i}$	angles between the averaged direction $\hat{1}$ and the normal $\mathbf{w}$ to the critical plane proposed by Łagoda et al., with $i = 1, \ldots, 4$
$\tilde{\delta}$	off-angle according to one of the expressions reported in Eqs. (11) and (12)
λ_n	n-th spectral moment, where n is a positive real number
ν_a	expected rate of occurrence of the counted equivalent stress cycles
ν_p	expected rate of occurrence of peaks of the counted equivalent stress cycles
ν_0^+	expected rate of zero-upcrossings of $\sigma_{z'}$
$\sigma_{af,-1}$	normal stress fatigue limit for fully reversed normal stress (loading ratio $R = -1$)
$\tau_{af,-1}$	shear stress fatigue limit for fully reversed shear stress (loading ratio $R = -1$)
ϕ, θ, ψ	Euler angles
$\hat{\phi}, \hat{\theta}, \hat{\psi}$	averaged Euler angles
ω	pulsation

Recently, Łagoda et al. [31–33] have proposed some relationships different from the original δ expression, and have estimated fatigue life of construction materials under *cyclic loading*. In the present paper, these relationships are implemented in the proposed criterion in order to estimate fatigue life of materials under *random loading*. The scope is to verify whether such expressions are able to improve the criterion in terms of lifetime evaluation.

Firstly, the theoretical framework of the criterion is presented. Note that the background theory on the frequency-based characterization of uniaxial and multiaxial random stresses as well as on uniaxial spectral methods for fatigue damage assessment can be found in Ref. [25]. Then, an application related to fatigue tests on steel round specimens subjected to a combination of random non-proportional bending and torsion [34] is discussed.

2. Theoretical framework of the proposed criterion

Fig. 1 shows the algorithm of the stress-based critical plane criterion here proposed for the frequency-based analysis related to High-Cycle Fatigue (HCF) random multiaxial loading. All the reported steps are discussed in the following Sections.

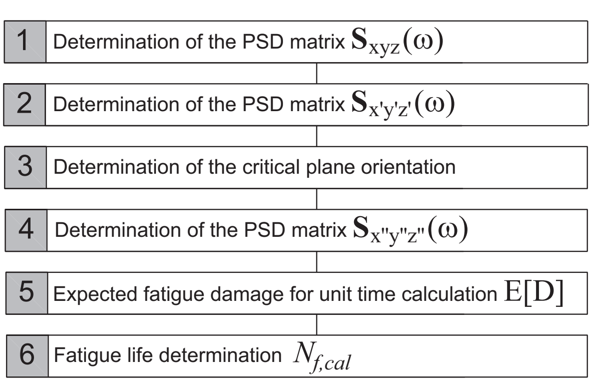

Fig. 1. Algorithm for fatigue life determination using the criterion herein proposed.

2.1. Determination of the PSD matrix with respect to the PXYZ reference system

Let us consider the stress tensor $\mathbf{s}_{xyz}(t) = \{s_1, s_2, s_3, s_4, s_5, s_6\}^T = \{\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}\}^T$ with respect to the fixed frame *XYZ* (Fig. 2a), at point *P* in the structural component exposed to a general time-varying random stress state. By assuming that the random features can be described by a six-dimensional ergodic stationary Gaussian stochastic process with zero mean values, the PSD matrix with respect to PXYZ (*Step 1* in Fig. 1) is here displayed:

$$\mathbf{S}_{xyz}(\omega) = \begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} & S_{1,4} & S_{1,5} & S_{1,6} \\ S_{2,1} & S_{2,2} & S_{2,3} & S_{2,4} & S_{2,5} & S_{2,6} \\ S_{3,1} & S_{3,2} & S_{3,3} & S_{3,4} & S_{3,5} & S_{3,6} \\ S_{4,1} & S_{4,2} & S_{4,3} & S_{4,4} & S_{4,5} & S_{4,6} \\ S_{5,1} & S_{5,2} & S_{5,3} & S_{5,4} & S_{5,5} & S_{5,6} \\ S_{6,1} & S_{6,2} & S_{6,3} & S_{6,4} & S_{6,5} & S_{6,6} \end{bmatrix} \quad (1)$$

where ω is the pulsation. The coefficients, $S_{i,j}(\omega)$, of such a matrix are defined by means of the auto/crosscorrelation functions, $R_{i,j}(\tau)$:

$$R_{i,j}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T s_i(t)\, s_j(t+\tau)\, dt \quad i,j = 1, \ldots, 6 \quad (2)$$

$$S_{i,j}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{i,j}(\tau) e^{-i\omega\tau} d\tau \quad i,j = 1, \ldots, 6 \quad (3)$$

being t = time and T = observation period.