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Kinematic synthesis using tree topologies

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ABSTRACT

This paper presents a methodology for the description and finite-position dimensional synthesis of articulated systems with multiple end-effectors. The articulated system is represented as a rooted tree graph. Graph and dimensional synthesis theories are applied to determine when exact finite-position synthesis can be performed on the tree structures by considering the motion for all the possible subgraphs. Several examples of tree topologies are presented and synthesized. This theory has an immediate application on the design of novel multi-fingered hands.

1. Introduction

Kinematic synthesis theory, in which an articulated system is designed to meet certain motion specifications, has been applied to open and closed linkages. In the dimensional synthesis problem, a given topology for an articulated system is dimensioned in order for its workspace to fit a given task. Finite-position dimensional synthesis of planar linkages was developed early, see [1–4]. Dimensional synthesis of spatial articulated systems has targeted mostly serial chains. The first methods used [5,6], based on geometric constraints and vector loop equations, were successful for synthesizing simple systems. More recently, methods based on robot kinematic equations [7,8] allowed to formulate design equations for more complex systems, still limiting the application to serial chains, and with limitations in the solution process [9]. Some examples of dimensional synthesis applied to parallel robots can be found in [10–12].

Type or structural synthesis [13,14] includes a systematic classification of the linkage type in the synthesis process, which is based on graph theory in many cases. Graph theory has been used for a long time in the analysis and type synthesis of linkages, see early research by Woo [15], Huang and Soni [16], Manolescu [17], Freudenstein and Maki [18], and more recently Tsai [19], Mruthyunjaya [20], and Lu, Mao et al. [21,22]. Additionally, Chuang and Lee [23] have used structural synthesis for the design of finger mechanisms.

The analysis of articulated systems with a tree structure has also generated some research. Selig [24] mentions tree-structured mechanisms and models them as rooted trees following [25]. His work includes basic definitions and the application to kinematic and dynamic analysis. Chen et al. [26] perform the analysis of tree-type geometries for applications in modular robots. Song and Amato [27] apply the analysis of tree-like articulated systems to folding. Jain [28] uses tree graphs for the dynamic analysis of multi-body systems; tree-systems also appear describing dynamic systems in Garcia de Jalon [29]. Tree articulated systems and their graph representation are also studied in [30] for the analysis and control of mechanical systems, where they are named forking linkages. Tischler et al. [31,32] apply graph theory for the structural synthesis of kinematic chains with applications to robot hands. However, no applications or methodology exist, to the knowledge of the authors, for the dimensional synthesis of articulated systems with a tree structure.

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Even though little literature is devoted to these systems when compared to their serial or parallel counterparts, tree articulated systems are widely used in robotic multi-fingered hands. Locomotive structures for mobile robots could also be modeled as tree-like systems. For this research, the kinematic design of multi-fingered robotic grippers lies in the primary application.

The advantage of having a methodology for the dimensional synthesis of a set of fingers is the possibility of defining simultaneous manipulation and grasping actions for the whole hand. Aside from underactuated robotic hands (see [33] for a review and [34] for recent results), which present little dexterity, the design of robotic hands has been performed from an anthropomorphic point of view. See recent reviews of different applications in [35,36]. In addition to this, modular fingers have been designed that are able to perform certain grasping actions and are later integrated in a hand [37]. The development of a theory for the use of dimensional kinematic synthesis on tree topologies aims to create a tool for new designs that can be applied, for instance, in many areas of human-robot interaction.

This paper focuses on developing a general methodology for the finite-position dimensional synthesis problem, in particular exact dimensional synthesis, applied to articulated systems with a tree structure. A tree-structured articulated system has a base (a grounded link), some common joints, and multiple end-effectors, each one of them corresponding to a separate branch.

The input data of the synthesis algorithm is a finite set of rigid-body positions for each of the multiple end-effectors and a selected tree topology, which is modeled using rooted tree graphs as explained in Section 2. After the substitution of kinematic chains, a compact rooted graph is obtained. This graph must be checked for solvability. In case of obtaining a non-solvable graph, an equivalent graph must be found. These two steps are developed in Sections 4 and 5.

For the resulting solvable rooted tree graph, the forward kinematics equations of relative displacements for each serial chain are computed using dual quaternions. The theory is introduced in Section 3. The synthesis of spatial serial chains for up to five degrees of freedom using this technique was developed in [38]. In this case, each serial chain corresponds to one branch, including the common joints. The dependency among the different serial chains allows the extension of the exact synthesis to articulated systems with a high number of degrees of freedom, and to tasks defined by a high number of positions. The extension of the kinematic synthesis to tree topologies is presented in Sections 6 and 7, and the matrix representation can be found in Section 8.

Each serial chain yields a set of equations and all the sets are solved simultaneously. Due to the high dimension and degree of the obtained system of equations, a numerical solver is required. As a last step, the solution is used to dimension the substituted kinematic chains.

Three examples are included in Section 9. The first one is a possible application to a multi-fingered robotic hand, for which numerical solutions have been found [39]. The second example is a simple PR-(R,P) tree structure, which can be solved analytically and is used to illustrate the process. The last example includes a numerical task for an RR-(RR,R,R) topology, and one of the multiple solutions obtained is presented.

For ease of comprehension, an overview of the nomenclature used in this paper is presented in Table 1.

The development of a dimensional synthesis method for tree articulated systems is a first step towards a new design tool for multi-fingered robotic hands. The non-anthropomorphic hands obtained, which can perform human tasks if so designed, could be used for specific robotics applications, and in particular for human–robot interaction or cooperative tasks.

2. Rooted tree graph representation

Articulated systems with a tree topology can be modeled using graph theory. This allows for a compact representation of the structure, the identification of key paths in the system, and the realization of some operations that help simplify the synthesis process.

The use of graphs in order to represent mechanisms was proposed by Crossley [40]. Tsai's methodology [19] is followed in this paper. It consists of identifying the joints with the edges and the links with the vertices of the graph. The different types of joints (revolute, prismatic, spherical, etc.) are indicated using their common abbreviation (R, P, S, etc.) on the edges of the graph. See for instance Fig. 1.

For their use in the kinematic synthesis process, the mechanisms are to be represented always as rooted graphs, the root vertex being fixed with respect to the reference system.

Table 1
Nomenclature used in this paper.

Symbol	Description
s	A vector.
$[M]$	A matrix.
$\hat{a}$	A dual number.
$\hat{Q}$	A dual quaternion.
S	Plucker coordinates of a line; also a screw.
$\$^*$	Smallest subalgebra of (3) containing all the possible infinitesimal mechanical liaison between two rigid bodies.
$\$$	An ordered screw surface.
$\mathfrak{L}$	Linkage locus space.