Theoretical and Applied Fracture Mechanics 81 (2016) 11–24

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

Boundary effect on crack kinking in a piezoelectric strip with a central crack

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ARTICLE INFO

Article history:
Available online 8 December 2015

Keywords:
Crack kinking
Piezoelectric strip
Integral transform
Dual integral equations
Hoop stress intensity factor

ABSTRACT

In this paper, the boundary effect on crack kinking in a piezoelectric strip with a central crack vertical to the boundary under in-plane loadings has been investigated. By using integral transform techniques the present mixed boundary value problem was reduced to the solution of dual integral equations, which can be further reduced to solving Fredholm integral equations. Analytical solutions can be obtained when the width of the piezoelectric strip are of infinite size. The asymptotic fields around crack tips have been provided for the electrically impermeable and permeable crack boundary conditions, respectively. Crack kinking phenomenon is investigated by applying the criterion of maximum hoop stress intensity factor. Numerical results show that the material properties, the geometry of the cracked strip and the electric-mechanical loading significantly affect the singular field distributions around the crack. Crack kinking is found for the impermeable crack case while no kinking for the permeable crack. Crack kinking angles for different piezoelectric materials under the impermeable crack surface condition have been predicted with certain loading and boundary conditions.

1. Introduction

Because of the intrinsic electromechanical coupling effect, piezoelectric materials have been used in actuators and transducers for a variety of applications. In view of their brittleness, piezoelectric materials have a tendency to develop critical cracks, which could inhibit their potential applications. In order to provide better understanding on the fracture behavior of piezoelectric materials, a vast body of literature has been developed [1–16]. Fracture analysis of a cracked functionally graded piezoelectric strip under anti-plane mechanical and inplane electric loading has been provided by Mousavi and Paavola [17] by using the distributed dislocation technique. The fracture problem for a medium composed of a cracked piezoelectric strip with functionally graded orthotropic coating was studied by Bagheri et al. [18] and the formulation is applicable for multiple cracks with arbitrary arrangement and different bonding coefficients. An analytical model for the analysis of an orthotropic strip with piezoelectric coating weakened by multiple defects has been proposed by Bayat et al. [19] using the dislocation method and the technique of singular integral equations. Although so much attention has been focused on the fracture analysis of piezoelectric materials, in which most researches are confined to the crack problems in infinite piezoelectric materials, less effort has been paid to study the fracture problems in piezoelectric solids of a finite size due to the complexity of the mathematical treatment. It is for this reason that we offer the current study to investigate the size effect on crack kinking in a cracked piezoelectric strip under in-plane mechanical and electric loadings.

Crack kinking is an important feature of fracture in brittle materials and several studies on crack kinking have been conducted [20–26]. By using a perturbation technique, an approximate description of crack kinking and curving under mixed loading and in the presence of in-plane stresses has been given by Karihaloo et al. [27]. Considering the intrinsic electro-mechanical coupling effect, the problem of crack kinking in piezoelectric materials has received much attention. McHenry and Koepke [28] reported the phenomenon of crack branching in piezoelectric ceramics under electro-mechanical loads based on their experimental study. Park and Sun [29] have also encountered the above-mentioned phenomenon in their experimental work. Zhu and Yang [30] modeled the crack kinking in a piezoelectric solid by a continuous distribution of edge dislocations and electric dipoles and the solution was formulated via the Stroh formalism. Qin and Mai [31] dealt with the problems of crack kinking in a piezoelectric biomaterial system with various material combinations and studied the effect of electric field on the path selection of crack extension. A moving mode-III crack at the interface between two dissimilar

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http://dx.doi.org/10.1016/j.tafmec.2015.11.007
0167-8442/

piezoelectric materials was studied by Li et al. [32] and their results show that the applied electric loading can disturb the stress field for a moving impermeable crack. Soh et al. [33] investigated the generalized plane problem of a moving Griffith crack in anisotropic piezoelectric solids based on the extended Stroh formalism and the crack branching is predicted. Hu and Zhong [34] considered a moving mode-III crack in a functionally graded piezoelectric strip, and found that the gradient of the material properties can affect the magnitudes of the stress intensity factors, and a high crack moving velocity may change the propagation orientation of the crack. Beom and Kang [35] investigated the crack kinking induced by domain switching in a ferroelectric material subjected to purely electric loading, and the size and shape of the switching zone has been estimated approximately by using the nonlinear electric theory. The size effect on crack kinking in a piezoelectric strip under impact loading was considered by Hu and Chen [36] and the results show that the geometry of the cracked strip and the electric loading significantly influence the singular field distributions around the crack tip and the crack kinking angle.

It is noted that the plane strain electroelastic problem of a cracked piezoelectric strip has been considered by Shindo et al. [37] using the permeable crack face conditions and the stress intensity factor and energy release rate for some piezoelectric ceramics are obtained, while the singular fields near the crack tip were expressed incorrectly. Wang and Mai [38] studied a cracked piezoelectric material strip under combining mechanical and electrical loads by using the technique of singular integral equations and obtained the intensity factors. The main focus of this paper is the asymptotic fields near the crack tip and the study of boundary effect and crack surface conditions on crack kinking of the cracked piezoelectric strip under in-plane mechanical and electric loadings, and this has not been reported in the literature, to the authors' best knowledge.

In this paper, the boundary effect on crack kinking in a piezoelectric strip with a central crack vertical to the boundary under in-plane loadings has been investigated following the method used in Chen and Meguid [12] in which the transient response of a piezoelectric strip with a central crack under anti-plane loading was studied. Fourier transforms is employed to reduce the mixed boundary value problem of the crack to solving a system of Fredholm integral equations. Both impermeable and permeable crack assumptions are considered. The asymptotic fields near the crack tip are obtained in an explicit form and the hoop and shear stress intensity factors are defined. Analytical solutions for the infinite cracked piezoelectric solid can be recovered if the size of the piezoelectric strip goes to infinity. The crack kinking phenomenon is investigated by applying the maximum hoop stress intensity factor criterion. The coupled electro-mechanical effects on the crack-tip fields are investigated. The influence of the geometric size of the piezoelectric strip on crack kinking is discussed.

2. Problem statement and method of solution

Consider a transversely isotropic, linear elastic piezoelectric material and denote the rectangular coordinates of a point by (x, y, z). In the absence of body forces and electric charge density, the equilibrium equations for plane strain piezoelectricity are

$$\begin{aligned} &C_{11}u_{x,xx} + C_{44}u_{x,zz} + (C_{13} + C_{44})u_{z,xz} + (e_{31} + e_{15})\phi_{,xz} = 0 \\ &(C_{13} + C_{44})u_{x,xz} + C_{44}u_{z,xx} + C_{33}u_{z,zz} + e_{15}\phi_{,xx} + e_{33}\phi_{,zz} = 0 \\ &(e_{31} + e_{15})u_{x,xz} + e_{15}u_{z,xx} + e_{33}u_{z,zz} - \lambda_{11}\phi_{,xx} - \lambda_{33}\phi_{,zz} = 0 \end{aligned} \tag{1}$$

where $u_x,\ u_z$ are components of the displacement vector and ϕ is the electric potential, $C_{11},\ C_{13},\ C_{33},\ C_{44}$ are elastic constants, $e_{15},\ e_{31}$ are piezoelectric constants, and $\lambda_{11},\ \lambda_{33}$ are dielectric permittivities.

The constitutive equations can be written as

$$\begin{aligned} &\begin{Bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{13} & 0 \\ C_{13} & C_{33} & 0 \\ 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ 2\varepsilon_{xz} \end{Bmatrix} - \begin{bmatrix} 0 & e_{31} \\ 0 & e_{33} \\ e_{15} & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_z \end{Bmatrix} \\ &\begin{Bmatrix} D_x \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & e_{15} \\ e_{31} & e_{33} & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ 2\varepsilon_{xz} \end{Bmatrix} + \begin{bmatrix} \lambda_{11} & 0 \\ 0 & \lambda_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_z \end{Bmatrix} \end{aligned} \tag{2}$$

where $\sigma_{ij},\ \varepsilon_{ij},\ D_i$ and $E_i\ (i,j = x,\ z)$ are components of stress, strain, electric displacement and electric field, respectively.

The gradient equations are

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad E_i = -\phi_{,i} \quad (i,j = x,z) \tag{3}$$

We study a Griffith crack of length $2c$ in a piezoelectric strip of width $2h$, with the poling direction perpendicular to the crack plane, as shown in Fig. 1. For convenience, a set of Cartesian coordinate system (x, y, z) is attached to the crack. Uniform normal stress, P_0, and electric displacement, D_0, or electric field, E_0 are applied on the far-field of the strip.

Considering that the electric permittivities of piezoelectric materials are on the order of a few thousand times the electric permittivity of air or vacuum inside the crack, the impermeable electric boundary condition on the crack face can be assumed. Only the right half part of the strip $(0 \leqslant x \leqslant h,\ z \geqslant 0)$ with appropriate boundary conditions needs to be analyzed owning to symmetry. The corresponding boundary conditions on the strip edges and the impermeable crack faces are:

$$\begin{aligned} &\sigma_{zz}(x, +\infty) = P_0 \\ &\sigma_{xz}(x, +\infty) = 0 \end{aligned} \qquad (0 \leqslant x \leqslant h) \tag{4.1}$$

$$D_z(x, +\infty) = D_0 \qquad (0 \leqslant x \leqslant h) \qquad (\text{Case } 1) \tag{4.2}$$

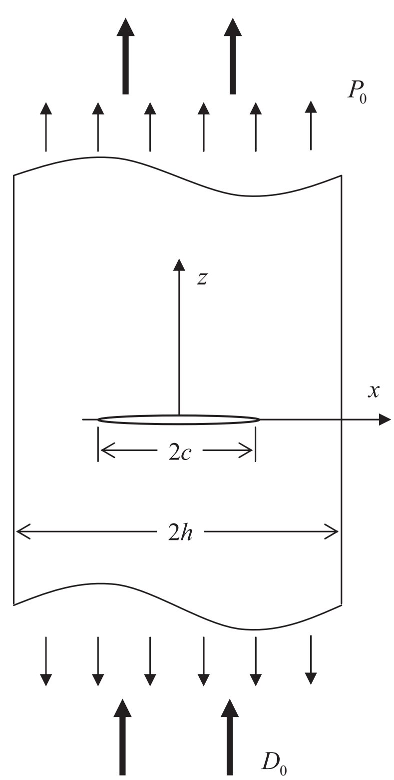

Fig. 1. Configuration and coordinate system for the cracked piezoelectric strip.