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A model for the study of meshing stiffness in spur gear transmissions

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ABSTRACT

This work describes an advanced model for the analysis of contact forces and deformations in spur gear transmissions. The deformation at each gear contact point is formulated as a combination of a global and a local term. The former is obtained by means of a finite element model and the latter is described by an analytical approach which is derived from Hertzian contact theory. Then the compatibility and complementary conditions are imposed, leading to a nonlinear system of equations subjected to inequality restrictions that should be solved once the position of each gear centre is known. A numerical example is presented where the quasi-static behaviour of a single stage spur gear transmission is discussed, showing the capabilities of the methodology to obtain the Loaded Transmission Error under several load levels as well as some other related measures such as load ratio or meshing stiffness.

1. Introduction

Gear transmissions are critical mechanical components found in a wide range of machinery. The industrial applications are countless and cover fields such as aerospace, agriculture or wind generation among others. As the working speeds for gear transmissions increase [1], the dynamic problems become more important and, as a consequence, the dynamic behaviour of gear transmission has also become a growing subject of concern for manufacturers and final users, involving aspects such as design, condition monitoring, vibration and noise control [2].

The main feature that characterises the dynamic behaviour is the periodic excitation induced by the variable meshing stiffness [3]. This phenomenon is a consequence of the changes in the number of teeth couples contacting simultaneously and it is a function of the angular position over the meshing period. Moreover, particularly in spur gears, the magnitude of the transmitted torque modifies the features of the meshing stiffness and therefore the dynamic behaviour. The characterisation of this periodic excitation is crucial in order to achieve better analysis in the design stage [4], improving durability but also reducing the levels of noise and vibration.

Gear noise and vibration are closely related with the so-called Transmission Error (TE), which was defined by Harris as "the difference between the position that the output shaft of a drive would occupy if the drive were perfect and the actual position of the output" as cited in ref. [5]. According to the previous definition, the TE should be expressed in radians but in order to make its understanding easier, TE in radians is actually multiplied by the base radius and provided in micrometers as a function of the angular position [3]. If gear teeth were perfect and non-deformable there would be no TE unless tooth profiles deviate from their theoretical shape. Nevertheless, real gears have flexible teeth and therefore the TE appears as a magnitude dependent on the load. At this point it should be stated that even though the term TE is widely used indiscriminately, it is possible to distinguish several kinds of TE depending on their source.

Thus, in real gears, profile and pitch errors are unavoidable and as a consequence of this, the so-called *manufacturing error* (*ME*) appears, which does not consider deflections. On the other hand, the term used is the *static transmission error* (*STE*), which depends on the magnitude of the torque transmitted by the gear pair and is sometimes called *loaded transmission error* (*LTE*). Finally from the point of view of dynamics, the suitable term is *dynamic transmission error* (*DTE*), which is also affected by the rotational speed.

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Nomenclature

TE	Transmission Error
ME	Manufacturing Error
STE	Static Transmission Error
LTE	Loaded Transmission Error
DTE	Dynamic Transmission Error
FE	Finite Element
LR	Load sharing Ratio
m	Gear normal module [mm]
φ	Pressure angle [deg]
ad	Addendum
dd	Deddendum
r_{p}	Tip rounding radio [mm]
h	Depth of the local Finite Element model [mm]
δ_i	Separation distance for the contact i, measured in the direction of the transmission line [mm].
C_{i0}	Centre of gear i.
d_0	Initial mounting distance
$\beta^k_{i,\,j}$	Flexibility of point j (at the radius R_j), located on the flank k when force is applied to the point i (on the radius R_i) in the active flank.
x_r ,y_r ,θ_r	Position and orientation of gear r
ρ_r	Involute base circle
ψ	Inclination of the centre axis
d_{T}	Working distance
φ_{T}	Pressure angle after gear centre displacements
Cr	Centre of the tip rounding arc
ρ_{2r2}	Equivalent base circle for contacts on the tip rounding
θ^*	Rotation with respect to the reference position for inverse contact
e_{10}	Tooth thickness in the initial pitch circle
φ_{Tr2}	Equivalent pressure angle for contacts on the tip rounding
N	Number of potential contact points
ε	Transverse contact ratio
u_{Tj}	Total displacement at the contact point j
F_i	Unitary force in the direction normal to the tooth profile at point i
n_{a}	Nodes on the active side
n_{r}	Nodes on the remaining edges
$(\lambda_{j,i})_{\mathrm{r}}$	Deflection for the gear r of the contact point j when a unitary force is applied at contact i obtained by interpolation from the corresponding flexibility matrix.
p	Load intensity over the thickness
L	Extension of the elliptic distribution of the pressure around the location of the load
$\{F_k\}$	Vector of unknown meshing forces
$\{q\}$	Generalised Coordinates of the gears
$[\lambda\ (q)]_N$	Flexibility matrix for N contacts corresponding to the location of gears defined by $\{q\}$
OLOA	Off-Line-Of-Action
LOA	Line-Of-Action
$F_{\mathrm{f}\ ri}$	friction forces at the contact i on gear r
f	Friction coefficient
$v_{Pi(1/2)}$	Relative velocity between the contacting points on each contacting surface
t	Unitary vector defining the common tangent of the surfaces
v_0	Threshold level to smooth the transition when the relative velocity is null
$\Omega_{(i/0)}$	Absolute rotational velocity for gear i
$v_{Ci(i/0)}$	Absolute velocity for gear centre i
C_iP_i	Vector from the gear centre C_i to the contact point P_i
$C_jQ^i_j$	Base radius for the contact point P_i on the gear j (equivalent base circle for contact *Involute – Tip rounding*)
$Q^i_jP_i$	Distance between the tangent point on the base circle and the contact point
T_{Ext}	Applied torque on gear 1
$\theta_{2,0}$	Initial value of the gear 2 angular position for iterative solution
$\Delta\theta_0$	Initial value of the transmission error used to start the numerical procedure
$K_{\mathrm{m}}\ (\theta_1,\ T_{\mathrm{Ext}})$	Meshing stiffness