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Interaction between the doubly periodic interfacial cracks in a layered periodic composite: Simulation by the method of singular integral equation

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ABSTRACT

In this paper, a doubly periodic interfacial cracking problem in a layered periodic composite is analyzed under antiplane shear loads. To further understand the interaction effect among the periodic interfacial cracks, four crack configurations are considered, namely a bilayered composite containing a single crack and a periodic array of collinear cracks, a layered periodic composite with a periodic array of parallel cracks and a doubly periodic rectangular array of cracks. In view of the periodic symmetry, the three periodic problems are transformed to a mixed boundary value problem for a single crack problem in an appropriate cell with the suitable periodic boundary conditions on its boundaries. The treatment skills for the periodic array of collinear cracks and the disposal techniques for the periodic array of parallel cracks are used together to solve the doubly periodic rectangular array of cracking problem. To demonstrate the computational accuracy of the present method, detailed comparative analyses are carried out. Numerical results are presented to show the interaction effect among the periodic interfacial cracks. For the doubly periodic rectangular array of cracking problem, parametric studies on the stress intensity factors leads out the following rule: the shielding effect of multiple parallel cracks and the amplifying effect of multiple collinear cracks exist simultaneously, and a coupled effect between geometrical and physical parameters on the interfacial fracture behavior exists clearly.

1. Introduction

Many engineering structures are made by bonding together periodically two or more material layers with different material properties [1]. A new class of functional composites, the so-called phononic crystals (PCs), consists of two or more different constituent materials with a periodic arrangement, and many anomalous wave propagation phenomena may arise in such periodic structures. The wave propagation phenomena, such as band gaps and localization phenomena, have potential applications such as acoustic filters, vibration control and isolation, noise suppression and design of novel acoustic devices [2,3]. Additionally, the bodies with microperiodic stratified structures can be found in nature (waved clays, sandstone-slates, sandstone-shales, thin-layered limestones). Thus, the problems of layered periodic composites are very important in engineering and geophysics.

Analysis of the layered periodic composite has been intensively studied in literature. Mal [4] presented a matrix method to solve wave propagation problems in multilayered anisotropic media subjected to time harmonic disturbances. Pang et al. [5] were concerned with wave propagation and localization problem in piezoelectric and piezomagnetic layered periodic structures. Liu et al. [6] investigated the shear horizontal waves propagating in a layered periodic structure that consists of piezoelectric layers perfectly bonded with piezomagnetic layers alternately. The Floquet-type elastic waves for an elastic composite with periodic microstructure are analyzed by Nemat-Nasser et al. [3]. Time-harmonic plane elastic SH-waves propagating in periodically laminated composites with functionally graded interlayers are investigated by Golub et al. [7].

In the above-mentioned analyses for layered periodic composite, laminar bonding was assumed perfect, that is, continuity conditions in elastic fields at interfaces are satisfied. In practice, however, the manufacturing techniques or aging of adhesive layers may lead to the initiation and growth of microcracks or cavities, thus reducing the bonding strength and resulting in possible

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interfacial slip and debonding. Many models including a set of micro-cracks, a thin visco-elastic layer, and a spring boundary condition are performed to model such damage for the layered periodic composite [8–12].

Schematically illustrated in Fig. 1 is a layered periodic composite with the damaged interface, in which the damaged interface was approximated by a doubly periodic array of cracks [10]. A great quantity of research work on the interaction of the doubly periodic cracks has been performed. Using the certain distribution of dislocations, an infinite elastic solid containing a doubly periodic rectangular array of slitlike cracks is considered by Delameter et al. [13]. Karihaloo [14] studied the stress relaxation process from the tips of doubly-periodic (rectangular and diamond-shaped) arrays of slit-like cracks contained in an infinite elastic solid under both plane and anti-plane strain conditions. Based on boundary collocation method, Isida et al. [15] obtained numerical solutions to the problem of doubly periodic rectangular array of model I cracks. Sahasakmontri et al. [16] presented a superposition approach for studying the influence of bridging forces upon the opening of multiple cracks in elastic solids under unidirectional tensile loading. Karihaloo et al. [17] presented an accurate and efficient method for computing the interaction of a set of or multiple sets of general doubly periodic cracks. A simple and robust method has been proposed to investigate doubly periodic array of cracks in an infinite plate by Chen and Lee [18]. Wang [19] analyzed the interaction of multiple rows of periodic cracks by using the superposition principle, pseudo-traction method and isolating analysis technique. The boundary integral equation approaches are used to study the doubly periodic array of cracks/rigid-line inclusions in an infinite isotropic plane medium by Dong and Lee [20]. Recently, by combining the elliptical function theory and conformal mapping technique, a closed form solution for the doubly periodic cracks in piezoelectric materials under far-field antiplane mechanical load and inplane electric load is derived by Tong et al. [21].

In comparison with the studies mentioned above, however, the doubly periodic cracking problem for the layered periodic composite received little attention. Some research works mentioned above are based on the complex variables method, because its governing equations are biharmonic equation or Laplace equation which solution can be expressed by analytic functions. Hence, more

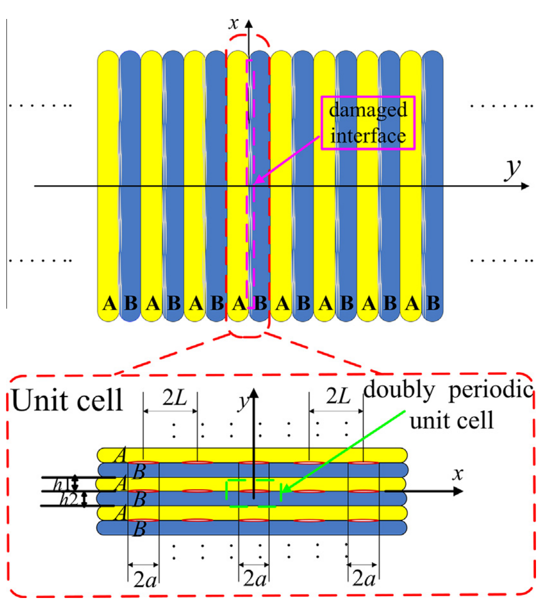

Fig. 1. Mathematical models of a layered periodic composite with a damaged interface which is approximated by a doubly periodic array of cracks.

remarkable, these methods mentioned above have not fully suited for analyses of the doubly periodic cracking problem for the layered periodic composite due to the existence of piecewise governing equations for each of the dispersion phases included the composites. Forty years ago, Erdogan and Gupta [22] used the dislocation technique to discuss the single crack problem for the multi-layered medium. The dislocation technique with the superposition principle can solve the doubly periodic cracking problems in principle. In the framework of continuously distributed dislocation model, the appearance of double infinite summation has been frequently considered as the most troublesome problem. It is because the errors of double infinite summation can reduce the calculation precision. Karihaloo et al. [17] used a basic solution for a row of cracks to raise the simulate precision. But the similar basic solution does not exist for the layered periodic composite, which may results in the precision problem of the double infinite summation if one studies the doubly periodic cracking problem for the layered periodic composite by the previous superposition principle.

It is easy to understand that there exist two types of interference effect caused by multiple parallel cracks or multiple collinear cracks in the doubly periodic cracking problem. These interaction laws between parallel cracks or collinear cracks have drawn much attention of researchers in the past years [23–32]. Chen [23] considered a static and dynamic anti-plane problem for a periodic array of parallel cracks in a coating-substrate structure, and his solutions show that stress intensity factor decreases with decreasing the vertical spacing of parallel cracks. For the periodic collinear cracks in a coating-substrate structure, the study of Ding and Li [24] confirmed that stress intensity factor decreases as the horizontal gap of collinear cracks increases. These mean that there exist a shielding effect of parallel cracks and the amplifying effect of collinear cracks. Recently, Wan et al. [25] proposed a parallel interface cracks model for a layered periodic piezomagnetic/piezoelectric composite. For this model, there exists a nonmonotonic change in the curve of stress intensity factor versus crack vertical spacing since the certain electromagnetic coupling effect exists between layers of the periodic piezomagnetic/piezoelectric composite. Li and his co-authors [26–32] presented a series of studies to explicate the effect of multiple cracks interactions for all types of crack models and emphasized particularly the interfacial properties which have an important influence to the fracture properties. Li and Lee [26] proposed real fundamental solutions for in-plane magnetoelectroelastic governing equations firstly, and then investigated a collinear unequal cracks model which is more practical than the commonly-used collinear equal cracks model. For the anti-plane case, Li et al. [27] studied the influences of electromagnetic effects on the fracture behavior of two coaxial cracks parallel to the interface and each in a layer of the piezoelectric–piezomagnetic bi-layer structure. A bi-layered multiferroic composite with arbitrary number of interfacial collinear cracks is investigated under magnetostriction or electrostriction by Li et al. [28]. Further, Li et al. [29] promoted the parallel crack research forward to a mixed mode crack in a bi-layer multiferroic structure. In addition, one can note that the effect of multiple parallel cracks and the effect of multiple collinear cracks are included in the two coaxial interfacial multiple cracks model for a tri-layered multiferroic semi-cylinder which is proposed and analyzed by Xiong et al. [30]. As mentioned before, the interfacial damage for the layered composite also can be described as the interfacial imperfection coupling model, such as spring interfacial condition. Recently, Li et al. [31,32] began to explore the application of interfacial imperfection coupling model for the facture problem of a layered multiferroic composite.

The present work constitutes a continuing research in questing for a deep understanding of interesting multiple cracks interaction