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Design criterion for rubbery parts under biaxial loading

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ABSTRACT

No relevant criterion exists for designing elastomer parts under biaxial loading. Thus, in this work, we attempt to derive a design criterion for rubber-like materials under quasi-static plane stress loading. A set of experimental data published elsewhere is used, obtained from tests including various loading modes and performed on four materials.

In this study, we developed a theoretical approach based on a reference axis change principle, introduced earlier for solid composite propergol and elastomers. The designing criterion based upon equivalent elongation, evaluated in a polar coordinates system, is expressed as function of two parameters. The comparison of the failure experimental data with the prediction of our criterion, for several tested elastomers, shows a good agreement.

1. Introduction

Because of their mechanical performance, elastomers are very widespread in industry. Therefore, for engineers, a design criterion is required in order to predict and prevent the failure of the rubbery parts in a structure.

In the literature, fracture criteria of rubber-like materials can be classified into two main categories. The first one is based upon fracture mechanics concepts and deals with the complete fracture of a material containing a crack [1–4]. The second category assumes a perfect material and focuses on mechanical quantities governing the fracture. Indeed, in the absence of cracks, several authors [5–13] have developed the concept of failure envelope for rubbers, based on stress or elongation quantities. Later, Kawabata [14] found that the maximum elongation at break is constant under several biaxial loadings for two vulcanized elastomers. Another interesting elongation criterion is the one proposed by Boulenouar et al. [15] to characterize the failure of elastomers.

In our recent works [16,17], some fracture criteria were proposed and applied on unfilled/filled vulcanizates and thermoplastic elastomers under quasi-static plane stress loadings. Good agreement was highlighted when comparing the experimental data with theoretical predictions.

In the present paper, we continue our investigation on the fracture of non-cracked specimens using our experimental data [16,17]. An original concept is developed and a relevant design criterion is identified for various loading paths. The most important result refers to better results achieved with the proposed criterion compared with those of the literature. Moreover, this criterion is based on an equivalent elongation and requires only two parameters to be defined.

2. Preliminary study

Four kinds of materials NR, SBR, PU and TPE have been investigated. The mechanical behavior of the two vulcanizates NR and SBR is hyperelastic while PU and TPE materials behave like viscoplastic solids. To obtain a wide range of loading conditions, sets of experiments including uniaxial tension, equal-biaxial tension, biaxial tension with three ratios (1/2, 1/3 and 1/5) and pure shear were carried out.

2.1. Definition of the biaxiality ratio

In the principal directions, the deformation gradient tensor F of an incompressible material, $J = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 1$, can be written as the following diagonal matrix noted “diag”:

$$\bar{\bar{F}} = diag(\lambda_1, \lambda_2, \lambda_3) = diag(\lambda, \lambda^n, \lambda^{-(1+n)}) \tag{1}$$

where λ_i ($i = 1, 2, 3$) is the elongation in the principal direction “i” and n defines the biaxiality ratio.

At the break point, for the tested materials and each loading path, the obtained values of n are reported in Table 1.

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Table 1
Biaxiality ratio n at ultimate elongation.

Biaxiality ratio (n)	NR	SBR	PU	TPE
Uniaxial Tension (UT)	-0.5	-0.5	-0.5	-0.5
Pure Shear (PS)	0	0	–	–
Equal Biaxial Tension (ET)	1	1	1	1
Biaxial Tension (BT_1/2)	0.88	0.88	0.94	0.9
Biaxial Tension (BT_1/3)	0.81	0.82	0.89	0.82
Biaxial Tension (BT_1/5)	0.82	0.68	–	0.76

In what follows, we focus on the investigation of a rubber design criterion based upon ultimate principal extensions. So, as a first step, the fracture data obtained by Hamdi et al. [16,17] and Kawabata [14] will be examined. Then, the thorough analysis of these experimental results will lead us to propose a new fracture criterion.

2.2. Failure analysis in the (λ_i, λ_j) plan

When considering our previous experimental results [16,17] and the ones obtained by Kawabata [14], one can imagine two fictitious materials with extreme failure behaviors. Indeed, two cases of "insensitive" or "sensitive" materials, relative to the multiaxial effect, are defined. Thus, we have chosen various loading paths ; i.e. uniaxial tension (UT), equal biaxial tension (ET), pure shear (PS) and three biaxialités ratios BT_1, BT_2 and BT_3.

In Fig. 1a and b, the ultimate principal stretches λ_1 are plotted versus λ_2 in the elongation plan (λ_i, λ_j), where i and $j = 1 \ldots 2$ denote the principal directions and $\lambda_{UT} = 6$ (value taken as an

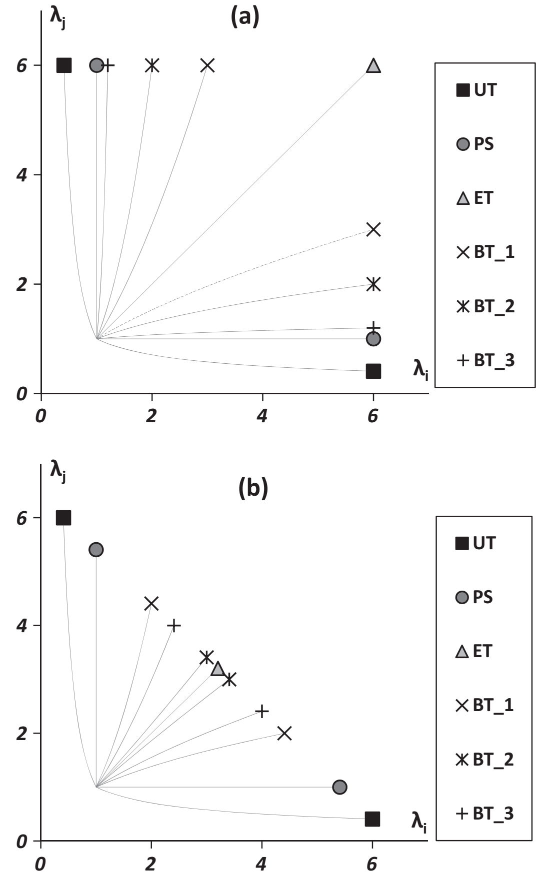

Fig. 1. Fracture data for the two fictitious materials: (a) «insensitive» and (b) «sensitive» materials.

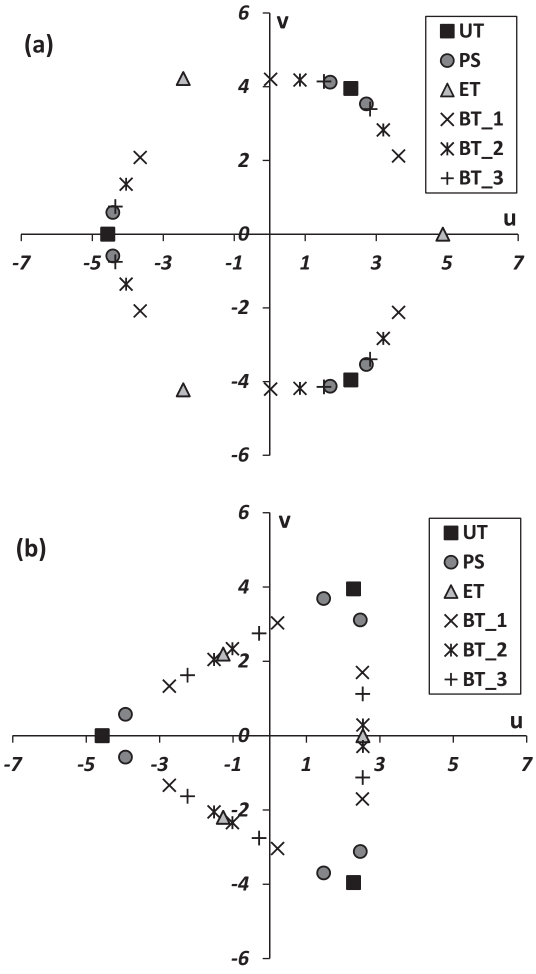

Fig. 2. Projection of the fracture data in (u, v) plan: (a) «insensitive» and (b) «sensitive» materials.

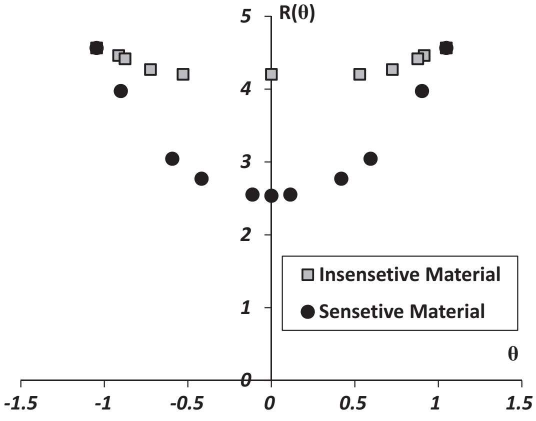

Fig. 3. Fracture data of the two fictitious materials in $(\theta, R(\theta))$ coordinates.

Table 2
R_c and K parameters of the trigonometric design criterion.

Material constants	R_c	K
NR	1.890	0.820
SBR	1.730	0.774
PU	2.314	0.859
TPE	2.190	1.335