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Effects of centrifugal load on tooth contact stresses and bending stresses of thin-rimmed spur gears with inclined webs

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ABSTRACT

Firstly, centrifugal deformation and stresses of thin-rimmed spur gears with inclined webs are analyzed in this paper with the finite element method (FEM) when the gears run at very high speeds. It was found that centrifugal load has a significant effect on tooth root bending stresses of the thin-rimmed gears at high speeds. So, it is necessary to consider the effect of the centrifugal load on tooth root bending stresses into bending strength calculations of the thin-rimmed gears when the gear speed is greater than 10,000 min^{-1}. Secondly, loaded tooth contact analyses (LTCA) are conducted for the thin-rimmed inclined web gears when they are deformed by the centrifugal load and engaged with a solid mating gear. FEM combined with a mathematical programming method is used to do the analyses. Tooth contact stresses and root bending stresses of the thin-rimmed gears are calculated under a torque load. It was found that the centrifugal deformation of the thin-rimmed inclined web gears almost has no effects on the tooth contact stresses and root bending stresses. This conclusion is different from the one obtained from the thin-rimmed straight web gears. Li (2008) [5] reported that the centrifugal deformation of the thin-rimmed straight web gears had significant effects on the tooth contact stresses and root bending stresses of the contact teeth. Finally, it is explained why the centrifugal deformation of the thin-rimmed inclined web gears does not affect the tooth contact stresses and root bending stresses.

1. Introduction

The thin-rimmed gears are often used in aircrafts to meet the need of lightweight [1,2]. Some of these gears have inclined webs for the sake of compact structure design [2]. Except for the lightweight, another characteristic of the thin-rimmed gears used in the aircrafts is the high-speed, for an example, higher than 10,000 min^{-1}. In this case, the effect of the centrifugal load on tooth surface contact strength and root bending strength of the gears must be considered.

So far, there have been a lot of researches conducted for various kinds of solid gears, but very a few for the strength calculations of the thin-rimmed gears. It is particularly difficult to find a paper on strength calculations of the thin-rimmed gears with inclined webs. Of course, it has been remained as an unsolved problem how to conduct strength calculations of the thin-rimmed inclined web gears used at high speeds.

Liu [3] discussed the effect of the centrifugal load on deformation and stresses of two thin-walled gears mounted on the two ends of a hollow shaft. Lewicki [4] discussed the effect of the centrifugal load on crack propagation direction of the thin-rimmed spur gears. Li [5] discussed the effect of the centrifugal load on strength calculations of the thin-rimmed spur gears with the straight webs, but the effect of the centrifugal load on the strength calculations of the thin-rimmed gears with the inclined webs has not been discussed. So, this paper tries to solve the strength calculation problems of the thin-rimmed inclined web gears used at high speeds.

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In this paper, three types of thin-rimmed spur gears with inclined webs on the left side of the tooth, the center of the tooth and the right side of the tooth separately are used as research objects. Firstly, deformation and stresses of these gears are analyzed under the centrifugal loads with FEM at the speed range of 5000–40,000 min^{-1}. It was found that web's centrifugal deformation of the thin-rimmed inclined web gears is different from that of the thin-rimmed straight web gears. This difference results in the new finding in the paper that the tooth contact patterns and contact stresses of the thin-rimmed inclined web gears are different from that of the thin-rimmed straight web gears when they are loaded under a torque and used at high speeds. It was also found that great tensile stresses occurred at the tooth roots of the thin-rimmed inclined web gears as well as the thin-rimmed straight web gears when the gear speed exceeds 10,000 min^{-1}. So, it is necessary to consider the effect of the centrifugal load in bending strength calculations of the thin-rimmed inclined web gears used at high speeds.

On the other hand, in order to investigate the effect of the centrifugal deformation on the tooth contact patterns, loaded tooth contact analyses are conducted for the thin-rimmed inclined web gears deformed by the centrifugal loads when these gears are engaged with a solid mating gear. FEM combined with the mathematical programming method is used to do the analyses [5,9–12]. It was found that the centrifugal deformation of the thin-rimmed inclined web gears almost has no effects on the maximum contact stress and the bending stresses of the contact teeth even if the gear speed is very high. This result is completely different from that of the thin-rimmed straight web gears [5].

Finally, it is concluded that the effect of centrifugal load on tooth bending strength of the thin-rimmed inclined web gears must be considered in strength calculations when the gear speed exceeds 10,000 min^{-1}. It is also pointed out that the effects of the centrifugal deformation of the thin-rimmed inclined web gears on the tooth contact pattern and tooth load distribution can be neglected in the strength calculations even if the gear speed is very high.

2. Deformation and stress analyses of the thin-rimmed inclined web gears under centrifugal loads

FEM is used to analyze deformation and stresses of the thin-rimmed inclined web gears under the centrifugal load condition for the sake of sophisticated structure of the thin-rimmed gears. When to perform the FEM calculations, it is necessary to make FEM models of the thin-rimmed inclined web gears, divide element meshes and form the centrifugal loads of all the nodes of the FEM models at the first. FEM models and the mesh-dividing are made automatically by software self-developed for these gears. Centrifugal loads on all the nodes are calculated in the following way.

The centrifugal load of every element is calculated as a distribution load on the element. This distribution load on the element is denoted by $\{P\}^e$ and it can be calculated using a formula of the centrifugal load [5]. Here, e is the element number. $\{P\}^e$ can be converted to nodal loads of the element e (denoted by $\{R\}^e$) based on the theory of FEM [6,7] using Eq. (1) in the following. Here, $[N]$ is the shape function matrix of the type of element used in the FEM models. The centrifugal loads on all the nodes of the FEM models can be obtained by assembling $\{R\}^e$ of all the elements together using the Eq. (2). Here, $\{R\}$ is the total centrifugal load array of the FEM models.

$$\{R\}^e = \iiint_V [N]^T \{P\}^e dV \tag{1}$$

$$\{R\} = \sum_{e=1}^{n} \{R\}^e \tag{2}$$

$$[K]\{\delta\} = \{R\} \tag{3}$$

$$[\sigma] = [D][B]\{\delta\}^e. \tag{4}$$

The centrifugal displacement $\{\delta\}$ of the nodes of the FEM models can be calculated through solving the Eq. (3). In the Eq. (3), $[K]$ is the stiffness matrix of the thin-rimmed gears. $[K]$ can be formed by FEM [6,7]. After the Eq. (3) is solved, the centrifugal displacement array $\{\delta\}$ can be obtained and the centrifugal stresses of the thin-rimmed gears can be calculated also using Eq. (4). Here, $[D]$ is the stress–strain matrix. $[B]$ is the strain–displacement transformation matrix [6,7].

FEM software was developed successfully for deformation and stress analyses of the thin-rimmed gears under centrifugal loads. Also, it was confirmed that the developed FEM software can make correct calculations of the centrifugal deformation and stresses of the gears in the previous research [5].

3. Loaded tooth contact analyses of the thin-rimmed inclined gears deformed by the centrifugal loads

In order to investigate the effect of the centrifugal load on tooth contact stresses and root bending stresses, LTCA are conducted for the thin-rimmed inclined web gears deformed by the centrifugal loads when the deformed gears are engaged with a solid mating gear. Since the centrifugal deformation of the solid mating gear is very small, it is neglected in the LTCA. Only the centrifugal deformation of the thin-rimmed gears is considered in the LTCA. Tooth load distributions can be obtained through the LTCA. Then tooth contact stresses and root bending stresses of the deformed gears can be calculated using FEM and the tooth loads obtained. FEM programs were successfully developed to do the analyses [8–11]. Reliability of the FEM programs was also confirmed in the previous researches [8–11]. Simple principle of the LTCA for the gears is stated also in the following.