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# Studying the load carrying capacity of spur gear tooth flanks

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ABSTRACT

The impact of load distribution in meshed teeth, teeth geometry and manufacturing accuracy on wear of the spur gear tooth flanks is studied in this paper. The original geometry of teeth is impaired wearing. Due to that, the load distribution is uneven, dynamic forces are increased, and energy efficiency is decreased. The aim of the theoretical and experimental studies, carried out in this paper, is to reach more accurate model for the analysis of tooth flanks load carrying capacity, taking into account a larger number of impacts on the tooth flanks stress state. A mathematical model for the contact stress during contact period is developed, depending on the value and sign of base pitch difference of meshed teeth. The impact of pitch point's position during the meshed teeth contact period was observed. In order to establish a correlation between the tooth flank failures and developed mathematical model, the appropriate experimental studies were carried out. The cemented spur gear pairs were examined on back-to-back gear test rig under conditions of maximum operational load. The results are presented in this paper.

## 1. Introduction

Train of gears generates the noise and vibrations and requires a great manufacturing accuracy. Due to the compact construction, high load carrying capacity and reliability, light weight per unit of transmitted power and small energy losses, gears have a wide range of application in various industries. Increasingly difficult economic, energy and environmental conditions in the world, require harsh operational conditions from the train of gears. In these operational conditions, the number of tooth flank failure forms has been significantly increased. There are more than twenty different forms of tooth flank failures according to ISO 10825 [1]. The dominant failures, i.e. failures that most commonly appear on the tooth flanks are adhesive wear, pitting and scoring. For that reason, these failures are the subject of the largest number of studies [1–14]. The influence of the pitch point on the emergence of scoring has been analysed by Imrek and Unuvar [2]. They found that conditions for scoring are the most unfavourable when pitch point is in the middle of contact period of teeth. The most favourable condition for scoring is when pitch point is maximally far away from the middle of contact period of teeth, i.e. when it is in the beginning or the end of tooth contact period. Based on the theoretical and experimental investigation by Vereš et al. [3], it is shown that scoring resistance of tooth flanks can be significantly improved using a special convex–concave profile in the plain tooth gears. Ristivojevic and Stefanovic analysed the influence of the pitch point's position of teeth on the emergence of pitting during contact period [4]. It was demonstrated that parts of tooth flanks below the pitch surface are much more susceptible to pitting than the parts of tooth flanks that are above the pitch surface. The impact of the intensity of spur gear tooth flanks wear on the change of contact stress was analysed by Flodin and Andersson [5] on the base of the developed numerical model. With the increase of tooth wear depth, the contact stress was changed due to the changes in the teeth profile geometry. Hence, this change is different in different points of teeth profile. The same authors have developed a model for monitoring the impact of tooth wear on tooth flank contact stress in cylindrical helical gear [6]. Larsson investigated the impact of oil type on the thickness of oil layer, contact stress and coefficient of friction [7]. It was shown that the oil type has significantly higher impact on the

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thickness of oil film and value of friction coefficient than the contact stress. Experimental research on the impact of oil amount in the lubrication system on the load tooth flanks carrying capacity is discussed by Höhn et al. [8]. In the field of small velocities, the oil amount does not have an expressed influence on the intensity of tooth wear. With the velocity increase, the impact of oil amount on tooth flanks wear is increased also. In addition, the impact gradient of oil amount on tooth wear is reduced when the velocity is larger than 0.57 m/s. In the study conducted by Pedrero et al. [9], a model of non-uniform load distribution along the line of contact of spur and helical gear teeth, obtained from the minimum elastic potential energy criterion, has been applied in the determination of the highest contact stress. Also a new design of cylindrical spur gears is suggested by Imrek [10] in which the tooth is wider in the single mesh zone than in the double mesh zone. By this design solution, the uniform load distribution along the teeth pair contact line during the contact period is obtained.

In this paper, the impact of the accuracy of teeth manufacturing and teeth geometry on load carrying capacity of tooth flanks is examined. The impact of manufacturing accuracy on the load carrying capacity of tooth flanks is discussed with respect to the value and the sign of base pitch difference of meshed teeth. The three cases were analysed: 1) the base pitches of simultaneously meshed teeth are equal, 2) the base pitch of driving gear is larger than base pitch of driven gear, and 3) the base pitch of the driven gear is larger than the base pitch of driving gear. This analysis is conducted through the developed model of load distribution in simultaneously meshed teeth. The impact of teeth geometry is observed through the tooth flank curve factor and through the load distribution factor. In the case of the observed cylindrical tooth pairs, by choosing the appropriate coefficients of profile shift, the position of pitch point during contact period was changed. In addition, the two cases were observed. The first case is standard, i.e. the pitch point is in the zone of single mesh, and in the second case the pitch point is located in the zone of double mesh. For this analysis, the appropriate mathematical models for the determination of the stress distribution on active surfaces of meshed teeth were developed. In the numerical example, the impact of geometrical and kinematical values of gear pair and load distribution on the tooth flank contact stress was studied. It was done by the application of the developed models. The appropriate experimental studies in the laboratory conditions were also conducted. The two groups of gear pairs were examined on a back-to-back gear test rig, in order to monitor the behaviour of tooth flanks under the conditions of maximum operation load. The examined groups of gear pairs had the same geometric and kinematic values like the gear pairs used in numerical example. The failures of tooth flanks were analysed with scanning electron microscope (SEM).

By comparative analysis of theoretical and experimental research, the correlation between the contact stress, load distribution, sliding speed and failures of tooth flank contact surfaces is established.

## 2. Theoretical considerations

### 2.1. Load distribution in simultaneously meshed teeth

The first step in the analysis of gear pair teeth operational ability from the aspect of bending and surface strength, noise, vibrations, efficiency, reliability and heating, is to determine the load distribution in simultaneously meshed teeth pairs. By engaging a larger number of tooth pairs in load transfer, the operational ability of gear pair teeth is increased. In general, the load distribution in simultaneously meshed teeth pairs is uneven. It means, that the simultaneously meshed teeth are differently engaged in load transfer of the gear pair.

This variation occurs due to inevitable form and dimension deviation in the manufacturing of teeth and due to elastic deformations of the teeth, body and crown of gears, shafts and supports. In addition, during the exploitation, due to the wear process, micro and macro geometries of tooth flanks are changed. For that reason, it is very difficult to provide a high degree of uniformity of load distribution in simultaneously meshed teeth pairs as well as immutability of distribution during the life cycle of gear pair.

Quantitative analysis of the participation degree of simultaneously meshed teeth pairs in transferring the total load of gear pair is conducted here through the factors of load distribution (Fig. 1):

$$K_{\alpha y} = \frac{F_y}{F} \tag{1}$$

where $F_y$ is the load transferred by the observed tooth pair and $F$ is the total load of gear pair.

The interval of load distribution factor values is $0 \le K_{\alpha y} \le 1$.

For describing the load distribution, the set of simultaneously meshed teeth pairs is observed as statically indeterminate system. From the conditions of elastic deformations and base pitch differences of simultaneously meshed teeth pairs, the appropriate mathematical models of load distribution, at characteristic contact points of meshed teeth (Fig. 1b), are formed. The first moment of the meshed teeth contact is between the dedendum of driving gear tooth (point $E_1$) and the top of driven gear point $A_2$ (Fig. 1a). In this moment of contact, the participation level of meshed teeth pair can be determined by the expression:

$$K_{\alpha \mathrm{E1A2}} = \frac{1}{1+\frac{c_{\mathrm{B1D2}}}{c_{\mathrm{E1A2}}}} \left(1 - \frac{c_{\mathrm{B1D2}}(p_{\mathrm{b1}} - p_{\mathrm{b2}})}{\frac{F_{\mathrm{t}}}{b}}\right) \tag{2}$$

where $p_{\mathrm{b1}}$ and $p_{\mathrm{b2}}$ are the real values of the base pitch of simultaneously meshed teeth pairs, $b$ is the face width, $c_{\mathrm{B1D2}}$ and $c_{\mathrm{E1A2}}$ are the equivalent specific stiffness of the meshed teeth at contact points $B_1D_2$ and $E_1A_2$, respectively and $F_{\mathrm{t}}$ is the tangential force in reference circle.