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Three dimensional fragmentation simulation of concrete structures with a nodally regularized meshfree method

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ABSTRACT

A three dimensional large deformation meshfree simulation of concrete fragmentation is presented by using a nodally regularized Galerkin meshfree method. This nodally regularized meshfree method is established with the two-level Lagrangian nodal gradient smoothing technique to relieve the material instability in failure modeling. The rate formulation is employed for the treatment of large deformation and therefore the two-level gradient smoothing is performed for the rate of deformation tensor and the deformation gradient. The essential characteristic of the present approach is that all the variables are conveniently computed at the meshfree nodes, which allows an efficient evaluation of the Galerkin weak form. The concrete failure is described by the KCC concrete model with three independent strength surfaces. This model has a pressure dependent evolving failure surface that is built with an internal damage variable. The computational implementation of the given concrete model within the context of meshfree formulation is discussed in detail. The effectiveness of the present method is demonstrated through several numerical examples of concrete structures.

1. Introduction

Concrete structure is one of the most frequently used engineering infrastructures and thus the modeling and analysis of concrete structures are of great importance [1]. The complex behaviors of concrete severely limit the analytical study and consequently the numerical modeling becomes a valuable and indispensable tool to investigate the concrete structures. A salient feature in concrete structural analysis is the brittle failure modeling [1]. One way for the failure modeling is to explicitly track the crack path, in present the extended finite element method [2,3] is one of the most popular approaches of this kind. Several meshfree and enriched/extended meshfree approaches [4–9] have also been introduced for explicit tracking of crack propagation. Alternatively, the failure may be implicitly treated by the phase-field modeling [10,11] or the damage mechanics formulation [12,13], among others. Here we employ the KCC concrete model with damage [14,15] to describe the concrete failure process since it can be conveniently implemented into a classical pressure-dependent elastoplasticity formulation. This model inherits an internal damage variable in its failure envelope and three independent strength surfaces are calibrated from the test data to model the failure evolution.

As for the computational concrete modeling, despite of the rapid development of various methods such as meshfree methods [16–18], extended finite element methods [2,3] and isogeometric analysis methods [4], we concentrate on the meshfree methods due to their global smoothing approximation, local refinement flexibility, and robustness in large deformation simulation [19–25]. The dynamic fracture in concrete was analyzed by Belytschko et al. [26] with the element free Galerkin method that was also used by Schwer et al. [27] to simulate the dynamic uniaxial tension test of concrete. A large deformation Lagrangian meshfree method was presented by Wu et al. [28] to deal with static modeling of geomaterials. Rabczuk and Eibl [29,31], Rabczuk et al. [30] and Rabczuk and Belytschko [32] carried out the high velocity concrete fragmentation analysis using the smoothed particle hydrodynamics and moving least square smoothed particle hydrodynamics methods. Meanwhile, Rabczuk et al. [33] and Rabczuk and Belytschko [34] have simulated the penetration/perforation on concrete structures as well with the element free Galerkin method, in which the residual velocity and energy balance were studied in detail. The meshfree simulations of plugging failures in high-speed

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impacts have been presented by Ren and Li [35]. A coupling of smoothed particle hydrodynamics and finite element methods was employed by Caleyron et al. [36] for reinforced concrete modeling which is also analyzed using a finite element material point method by Lian et al. [37]. Recently a meshfree meso–macro-multiscale method was proposed for concrete fracture analysis [38].

The meshfree methods we shall use are the Galerkin meshfree methods with moving least square [16] or reproducing kernel [17] approximations, which are equivalent for the monomial basis functions. One issue associated with the Galerkin meshfree methods is the high computational cost. Different methods have been proposed to improve the meshfree efficiency, particularly the nodal integration based meshfree methods. A direct nodal integration of the Galerkin weak form leads to the rank deficiency which can be corrected by adding residual terms [39] or stress points [40]. However these corrections may need artificial parameters or sacrifice the nodal integration nature. The stabilized conforming nodal integration (SCNI) developed by Chen et al. [41,42] bypasses this difficulty through a strain smoothing formulation [43], where the rank stability, linear exactness and nodal integration is uniformly achieved [44]. This approach has been successfully developed and generalized to many problems [45–52]. In case of fragment-impact problems, a semi-Lagrangian reproducing kernel particle method with SCNI and SNNI (stabilized nonconforming nodal integration) was proposed by Guan et al. [53], where extreme deformation and self-contact can be naturally dealt with. Wu et al. [54,55] developed a point-wise coupled reproducing kernel–finite element formulation to study the fragmentation and debris evolution process. In order to further regularize the material instability in damage modeling and resolve the discretization sensitivity issue in damage analysis, based upon the implicit regularization methodology developed by Chen et al. [43,62], Wang and Li [56] and Wang et al. [57] proposed a two-level strain smoothing meshfree approach within the stabilized conforming nodal integration framework [41,42].

In this work the two-level nodal strain smoothing formulation [56] is employed to develop a nodally regularized large deformation meshfree method for concrete failure simulation. The concrete failure is described by the pressure dependent KCC concrete model [14,15]. This model builds the damage into the evolution of failure surface which provides a significant computational convenience. Herein the concrete model is equipped with the rate formulation to deal with the large deformation effect. The two-level smoothed nodal rate of deformation tensor and the deformation gradient are systematically introduced into the objective integration of constitutive equations. The implementation of the concrete model is discussed in detail with particular reference to the update of state variables. All the variables are carried by the meshfree particles as offers an obvious computational advantage.

The organization of the rest of this paper is as follows. The Lagrangian meshfree approximation is briefly discussed in Section 2. In Section 3, the concrete equations are presented in detail with particular emphasis on the concrete model used in this study. In Section 4, the two-level smoothed nodal rate of deformation tensor and the deformation gradient are firstly introduced, as is followed by the nodally regularized meshfree discretization of the equation of motion with two-level smoothed measures. The capability of the proposed method is demonstrated in Section 5 through numerical examples. Finally Section 6 gives the conclusions of this work.

2. Lagrangian meshfree approximation

In this study the Lagrangian formulation is adopted and the initial problem domain is denoted by $\boldsymbol{X} \in \Omega_0$ that is mapped to the current configuration $\boldsymbol{x} \in \Omega$. In a Lagrangian meshfree approximation [18], the initial configuration Ω_0 is discretized by a set of meshfree particles $\{\boldsymbol{X}_I\}_{I=1}^{NP}$ and each particle $\boldsymbol{X}_I = (X_I, Y_I, Z_I)$ has a local support of $supp(\boldsymbol{X}_I)$ which is defined through the positive kernel function of $\psi_s(\boldsymbol{X}_I - \boldsymbol{X})$ such that $\cup_{I=1}^{NP}[supp(\boldsymbol{X}_I)] \supset \Omega_0$. In practice, the 3D kernel function $\psi_s(\boldsymbol{X}_I - \boldsymbol{X})$ is often constructed by the tensor product operation on its one dimensional counterparts in three dimensions, respectively, i.e.,

$$\psi_s(\boldsymbol{X}_I - \boldsymbol{X}) = \psi\left(\frac{|X_I - X|}{s_X}\right)\psi\left(\frac{|Y_I - Y|}{s_Y}\right)\psi\left(\frac{|Z_I - Z|}{s_Z}\right) \tag{1}$$

where s_X, s_Y and s_Z are the support sizes for each dimension. Here $\psi(\theta)$ is taken as the cubic B-spline function [22]:

$$\psi(\vartheta) = \begin{cases} \frac{2}{3} - 4\vartheta^2 + 4\vartheta^3 & |\vartheta| \leqslant \frac{1}{2} \\ \frac{4}{3} - 4\vartheta + 4\vartheta^2 - \frac{4}{3}\vartheta^3 & \frac{1}{2} < |\vartheta| \leqslant 1 \\ 0 & |\vartheta| > 1 \end{cases} \tag{2}$$

Then a meshfree approximant of a field variable $\upsilon(\boldsymbol{X})$, say $\upsilon^h(\boldsymbol{X})$, can be assumed to take the following form [16–18]:

$$\upsilon^h(\boldsymbol{X}) = \sum_{I \in SI(\boldsymbol{X})} \Psi_I(\boldsymbol{X})\upsilon_I \tag{3}$$

with

$$\Psi_I(\boldsymbol{X}) = \psi_s(\boldsymbol{X}_I - \boldsymbol{X})\boldsymbol{p}(\boldsymbol{X}_I - \boldsymbol{X})\boldsymbol{c}(\boldsymbol{X}) \tag{4}$$

where $\Psi_I(\boldsymbol{X})$ and υ_I are the meshfree shape function and nodal coefficient, $SI(\boldsymbol{X}) = \{I | \boldsymbol{X} \in supp(\boldsymbol{X}_I)\}$. $\boldsymbol{c}(\boldsymbol{X})$ is an unknown vector. $\boldsymbol{p}(\boldsymbol{X})$ is the n-th order monomial basis vector:

$$\boldsymbol{p}(\boldsymbol{X}) = \{1, X, Y, Z, X^2, \ldots, Z^n\} \tag{5}$$

Subsequently in order to solve $\boldsymbol{c}(\boldsymbol{X})$, the following n-th order reproducing conditions or consistency conditions are enforced.

$$\sum_{I \in SI(\boldsymbol{X})} \Psi_I(\boldsymbol{X}) X_I^{n_X} Y_I^{n_Y} Z_I^{n_Z} = X^{n_X} Y^{n_Y} Z^{n_Z}, \quad 0 \leqslant n_X + n_Y + n_Z \leqslant n \tag{6}$$

For convenience of expression, Eq. (6) can be equivalently cast into a vector form as:

$$\sum_{I \in SI(\boldsymbol{X})} \Psi_I(\boldsymbol{X})\boldsymbol{p}(\boldsymbol{X}_I - \boldsymbol{X}) = \boldsymbol{p}(0) \tag{7}$$

Thus substitution of Eq. (4) into (7) leads to

$$\boldsymbol{M}(\boldsymbol{X})\boldsymbol{c}(\boldsymbol{X}) = \boldsymbol{p}(0) \tag{8}$$

with

$$\boldsymbol{M}(\boldsymbol{X}) = \sum_{I \in SI(\boldsymbol{X})} \boldsymbol{p}(\boldsymbol{X}_I - \boldsymbol{X})\boldsymbol{p}^T(\boldsymbol{X}_I - \boldsymbol{X})\psi_s(\boldsymbol{X}_I - \boldsymbol{X}) \tag{9}$$

From Eq. (8) we solve:

$$\boldsymbol{c}(\boldsymbol{X}) = \boldsymbol{M}^{-1}(\boldsymbol{X})\boldsymbol{p}(0) \tag{10}$$

and consequently obtain the meshfree shape function $\Psi_I(\boldsymbol{X})$ as:

$$\Psi_I(\boldsymbol{X}) = \boldsymbol{p}^T(\boldsymbol{0})\boldsymbol{M}^{-1}(\boldsymbol{X})\boldsymbol{p}(\boldsymbol{X}_I - \boldsymbol{X})\psi_s(\boldsymbol{X}_I - \boldsymbol{X}) \tag{11}$$

3. Concrete equations

The motion of a concrete structure is governed by the following weak form via the updated Lagrangian formulation:

$$\int_\Omega \rho\delta\boldsymbol{u} \cdot \ddot{\boldsymbol{u}}\, d\Omega + \int_\Omega (\nabla_{\boldsymbol{x}}^S \delta\boldsymbol{u}) : \boldsymbol{\sigma}\, d\Omega - \int_\Omega \delta\boldsymbol{u} \cdot \boldsymbol{b}\, d\Omega - \int_{\Gamma^t} \delta\boldsymbol{u} \cdot \boldsymbol{t}\, d\Gamma = 0 \tag{12}$$

where ρ is the concrete density referring to the current configuration, $\boldsymbol{u}$ is the displacement vector, the overhead dot denotes the