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Thermally conducting collinear cracks engulfed by thermomechanical field in a material with orthotropy

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ABSTRACT

The problem of two collinear cracks in an orthotropic solid is investigated under applied mechanical and uniform heat flow loadings. The thermal medium crack model is applied to address the effects of the medium inside cracks. Applying the Fourier transform technique, the boundary-valve problem is reduced to solving triple integral equations, then to solving singular integral equations with the Cauchy kernel. The crack-tip thermoelastic fields involving of the strain energy density (SED) factors, the stress intensity factors, the jumps of temperature and elastic displacements across the cracks are given in closed forms. Numerical results are carried out to show the influences of applied mechanical loading and thermal conductivity of crack interior on the thermal stress intensity factors, the temperature change across crack faces and the strain energy density factors. The results reveal that the crack-tip thermoelastic fields are dependent on applied thermo-mechanical loadings and the thermophysical properties of crack interior. The crack-face thermal property is important and it should not be disregarded in analyzing thermoelastic problems of a cracked solid under a thermal loading.

1. Introduction

The intersections of heat and deformation have arisen in many engineering problems and attracted much interest [1,2]. One of the important topics is the thermal stress concentration near crack tips. To investigate the thermoelastic field around cracks, one can understand more about the stability and service life of cracked engineering materials and structures. Based on the theory of thermoelasticity [3], many efforts have been made for the thermoelastic analysis of a cracked solid. For example, the character of thermal-stress singularities at a crack tip has been investigated in [4] for a cracked transversely isotropic solid. The observations have revealed that the local singularities of thermal stresses near a crack tip are the same as those with mechanical stresses. Moreover, various two- or three-dimensional crack problems have been investigated under steady-state and transient thermal loadings respectively [5–13]. The considered solids are always assumed to be transversely isotropic, orthotropic and anisotropic respectively.

In the above mentioned works, the thermal boundary condition at the crack faces is always assumed to be thermally insulated or partially insulated. The influences of applied thermal loadings on the thermal stress intensity factors are often focused on. However, for a real situation, the crack may be filled with a certain medium, and it will affect the thermoelastic fields near crack tips [14]. Recently, considering the effects of thermophysical properties inside a crack, the thermal medium crack model has been proposed [15]. That is, the thermal flux on the crack face q_n^c is assumed to satisfy the following relation:

$$q_n^c = -\lambda_c \frac{\Delta\theta_n}{\Delta u_n}, \tag{1}$$

where λ_c is the thermal conductivity of the medium inside the crack; Δu_n denotes the jump of the elastic displacement across the crack; $\Delta\theta_n$ is the temperature difference across the crack. The thermally permeable and impermeable crack models are the limiting cases of the thermal medium one. Applying the boundary condition (1), the effects of applied mechanical loadings and thermal conductivity of crack interior on the thermal stress intensity factor can be addressed, which is different from the previous studies about the thermoelastic analysis of a cracked solid. And it has also been applied to deal with the opening crack problem in an orthotropic solid under thermomechanical loadings [16]. In addition, to characterize the thermal stress field, the thermal stress intensity factor is always calculated. When the uniform heat flow is applied past a crack, the obtained results show that the mode-II stress intensity factor is created [17]. The initiation of mode-II crack will be off the crack line microscopically and exhibit the micro and macro

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characters. Then the strain energy density theory is more suitable than stress intensity factor and energy release rate theory to predict the crack growth [18,19]. In the present paper, the strain energy density factor will be calculated and used to address the effects of the medium inside a crack.

On the other hand, to investigate the crack problem in an elastic solid, many mathematical methods have been used, such as the mapping techniques [20], the integral equation method [20,21], the linear sampling method [22], the Trefftz method [23] and so on. Note that the problem of two collinear cracks in an orthotropic material with heat flow loadings has been dealt with by using the series method [5]. However, the series solutions of the crack-tip thermal stress fields are only given when the symmetric heat flux is applied on the crack surfaces. And the effects of applied mechanical loading and thermal conductivity of crack interior on the thermal stress intensity factor are not considered. Here the boundary condition (1) is used to solve the problem of two collinear cracks in an orthotropic solid under thermomechanical loadings. Applying the Fourier transform technique and integral equation methods, we present the closed-form solutions of crack-tip thermoelastic fields. Numerical results show the dependence of the strain energy density factor, thermal stress intensity factor and the jumps of temperature change on applied mechanical loading and thermal conductivity of crack interior.

2. Problem formulation

As shown in Fig. 1, consider that two collinear cracks are embedded in an orthotropic solid. Cartesian coordinates system xoy is used and the cracks are located on $[-b,-a]$ and $[a,b]$, respectively. The uniform thermal flux $-q_0$ and mechanical loading $-\sigma_0$ are acting on the crack surfaces. For the state of plane stress, we have the following constitute equations [6]:

$$\sigma_{xx} = c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} - \beta_1\theta, \tag{2}$$

$$\sigma_{yy} = c_{12}\frac{\partial u}{\partial x} + c_{22}\frac{\partial v}{\partial y} - \beta_2\theta, \tag{3}$$

$$\sigma_{xy} = c_{66}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right], \tag{4}$$

with

$$c_{11} = \frac{E_{xx}}{1-\upsilon_{xy}\upsilon_{yx}}, c_{22} = \frac{E_{yy}}{1-\upsilon_{xy}\upsilon_{yx}}, \tag{5}$$

$$c_{12} = \frac{E_{xx}\upsilon_{yx}}{1-\upsilon_{xy}\upsilon_{yx}} = \frac{E_{yy}\upsilon_{xy}}{1-\upsilon_{xy}\upsilon_{yx}}, \tag{6}$$

$$\beta_1 = c_{11}\alpha_{xx} + c_{12}\alpha_{yy}, \beta_2 = c_{12}\alpha_{xx} + c_{22}\alpha_{yy}, \tag{7}$$

where u and v stand for the components of elastic displacement; θ is the temperature change; υ_{xx} and υ_{yy} are the poisson's ratios; E_{xx} and E_{yy} are the Young's moduli; $c_{66} = G_{xy}$ is the shear modulus; α_{xx} and α_{yy} are the coefficients of linear expansion.

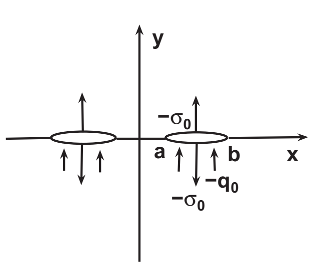

Fig. 1. Two collinear cracks embedded in an orthotropic solid under combined thermomechanical loadings.

Here we consider the stationary behavior of a cracked orthotropic solid. It is seen that although the steady-state analysis is highly idealized, the obtained results can be used to model some real-world phenomena [24]. Application of the elastic equilibrium equations leads to the governing differential equations as follows:

$$c_{11}\frac{\partial^2 u}{\partial x^2} + c_{66}\frac{\partial^2 u}{\partial y^2} + (c_{12}+c_{66})\frac{\partial^2 v}{\partial x\partial y} - \beta_1\frac{\partial\theta}{\partial x} = 0, \tag{8}$$

$$c_{66}\frac{\partial^2 v}{\partial x^2} + c_{22}\frac{\partial^2 v}{\partial y^2} + (c_{12}+c_{66})\frac{\partial^2 u}{\partial x\partial y} - \beta_2\frac{\partial\theta}{\partial y} = 0. \tag{9}$$

Moreover, Based on the theory of Fourier heat conduction, we have

$$q_x = -\lambda_x\frac{\partial\theta}{\partial x}, \quad q_y = -\lambda_y\frac{\partial\theta}{\partial y}, \tag{10}$$

where λ_x and λ_y are the thermal conductivities of an orthotropic material. From the thermal equilibrium equation, one obtains

$$\lambda^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0, \quad \lambda = \sqrt{\frac{\lambda_x}{\lambda_y}}. \tag{11}$$

In what follows, let us give the boundary conditions. Due to the symmetry of the considered problem, the thermoelastic fields in the region $x \geqslant 0$ and $y \geqslant 0$ are only investigated. In the present paper, using the thermal medium crack model, the crack-face boundary conditions can be written as

$$\sigma_{yy}(x,0) = -\sigma_0, \quad a < x < b, \tag{12}$$

$$\sigma_{xy}(x,0) = 0, \quad a < x < b, \tag{13}$$

$$q_y(x,0) = -(q_0 - q_c), \quad a < x < b, \tag{14}$$

where

$$q_c = -\lambda_c\frac{\theta(x,0)}{v(x,0)}, \quad a < x < b. \tag{15}$$

Moreover, the continuity of elastic displacements and temperature change along the crack-free parts of x-axis yields

$$v(x,0) = 0, \quad 0 < x < a, \quad x > b, \tag{16}$$

$$u(x,0) = 0, \quad 0 < x < a, \quad x > b, \tag{17}$$

$$\theta(x,0) = 0, \quad 0 < x < a, \quad x > b. \tag{18}$$

3. Solution procedure

Now let us solve the partial differential Eqs. (8), (9) and (11) with the boundary conditions (12)–(18). Since the thermal Eq. (11) is not coupled with the elastic strain, the temperature field will be solved independently.

3.1. Temperature field

Since the temperature change will be vanished for $y \to +\infty$, applying the Fourier transform technique, the solution of (11) can be expressed as the following integral

$$\theta(x,y) = \int_0^\infty A(\xi)e^{-\xi\lambda y}\cos(\xi x)d\xi, \tag{19}$$

where $A(\xi)$ is the unknown to be solved. From (10), one can arrive at

$$q_x(x,y) = \lambda_x\int_0^\infty \xi A(\xi)e^{-\xi\lambda y}\sin(\xi x)d\xi, \tag{20}$$

$$q_y(x,y) = \lambda_y\lambda\int_0^\infty \xi A(\xi)e^{-\xi\lambda y}\cos(\xi x)d\xi. \tag{21}$$

From the boundary conditions (14) and (18), we have the following triple integral equations