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Fast unconditionally stable 2-D analysis of non-conjugate gear contacts using an explicit formulation of the meshing equations

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ABSTRACT

Computerised analysis of the contact of gear teeth is currently dependent on numerical solution techniques involving implicit multi-equation systems. These present inherent convergence problems when the initial values are not close enough to the real solution and require significant computational effort. Here a comprehensive new solution is presented using a modified form for the fundamental gear meshing equations in two dimensions. This formulation allows the analytical reduction of the system of meshing equations to a single scalar equation, which is solved using a fast unconditionally stable numerical method. The need for careful determination of initial values for the numerical solution is eliminated and test runs on real gear geometries verify solution accuracy, stability and speed. Application of the algorithm to profile-modified involute gears and Geneva-type mechanisms and related results are shown.

1. Introduction

Practical gear applications often deviate from the theoretical conjugate forms (based on the involute, cycloidal, or other curves) either due to manufacturing errors, or to deliberate profile modifications [1]. Furthermore, a wide range of mechanisms and some gear forms, such as the Geneva gears [2] and the Wildhaber–Novikov gears are fundamentally non-conjugate. As a result, conventional gear theory the well-established theory in [3] cannot be used effectively in these contexts for gear contact and meshing. Mesh analysis of non-conjugate gears is important, because even minor modifications of the tooth form result in appreciable changes in the path of contact and the distribution of transmission errors. Such transmission errors are usually expressed as the deviation from a nominal angular gear position and their knowledge is vital for many applications, such as the dynamical simulation of gears and the prediction of dynamic loads [1].

The need to analyse non-conjugate meshing has led to the development of a few different analysis techniques, ranging from applied mathematical [4–8] to heuristic point-to-point simulation [9]. Other analysis techniques for fixed axis gear pairs rely on simplifications of geometry and report having some limitations in computational robustness [10–12]. Methods for general cam synthesis and optimisation are reported in [13,14], but have not been implemented to modified involute gears. Kinematics of intrinsically non-conjugate geometries, such as Geneva gears, is being actively researched and an analysis and design optimisation method specific (and limited) to these is discussed in [15].

Amidst the plethora of available methods and solutions, the most widely established general solution has been given in [4]. This solution is based on a basic set of the generalised equations for profile tangency and the two-dimensional problem yields a system of 4 implicit scalar equations (two of which are interdependent) in 3 unknowns plus a 4th unknown, which is usually the reference gear

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Nomenclature

θ_i	angular position of a tooth i
$\delta\theta_2$	transmission error
i_{12}	transmission ratio
r_f	inside (root) radius
r_k	outside radius
$\lVert\cdot\rVert$	Euclidian norm
$\mathbf{x}$	position vector
r_i	radius on profile of tooth i (profile parameter)
$\mathbf{p}_i$	vector function of tooth profile i
$\mathbf{R}_i$	rotation matrix around i-axis
$\hat{i}, \hat{j}, \hat{k}$	unitary vectors of ortho-normal coordinate system
ε	convergence tolerance for the numerical solution

position, acting as a parameter of the solution. To obtain non-trivial solutions, the Jacobian determinant of the system must be non-zero, therefore only 3 of the 4 interdependent equations must be chosen for the computation. Because they were concerned that an arbitrary choice of the 3rd independent equation presented accuracy concerns, Litvin et al. [5] later proposed a modified version of the meshing equations, resulting in the two-dimensional case in a system of 3 independent implicit equations in 3 unknowns. They observed in their implementation that a careful selection of initial values, or 'guess values', was needed, or the solution algorithm they employed [16] would not converge, terminating far from the actual solution; they reported solving this problem in special cases when the tooth form had been designed by a 'local synthesis' technique [4], prior to conducting the mesh analysis.

But what if local synthesis is not the design method for tooth geometry, i.e. in the general case of gears designed using different methods or gear forms derived from real measurements? Litvin et al. have proposed conducting parametric sweeps of the tooth profile independent coordinates, thereby locating appropriate initial values in an iterative trial and error fashion [17], and potentially involving many calculations depending on the parametric grid density.

This paper presents a new form for the equations of non-conjugate meshing, derived from a modified set of the generalised equations for profile tangency. These new equations are treated analytically and the solution process manages to reduce the system of 3 implicit equations in 3 unknowns to a single implicit equation in 1 unknown. Numerical solution of this reduced form is accomplished using a simple bisection algorithm, which is fast, does not rely on initial value estimates and is unconditionally stable. The other 2 unknowns are then calculated explicitly to complete the solution. Effectively, the definition field considered by the numerical solution is thus reduced from (a subset of) $\mathbb{R}^3$ to (a subset of) $\mathbb{R}$. Contact analysis runs are made on parameterised modified involute gear geometries and on a non-conjugate Geneva-type gear mechanism to verify solution accuracy, stability and speed.

2. Parametrisation of bearing surface profiles

In the context of the present study, contact profiles will be expressed parametrically as mappings of the form:

$$\mathbf{p} : \mathbb{R} \rightarrow \mathbb{R}^2.$$

In principle any mapping would be suitable, but in order to obtain explicit (even analytical) solutions, the mapping should be bijective. Here we consider the following mapping in the coordinate system *Oij*, as shown in Fig. 1, which originates at the centre of the gear and is fixed to it (thus rotating with the gear):

$$\begin{aligned} &\mathbf{p} : \mathbb{R}^+ \rightarrow \mathbb{R}^2 \\ &\mathbf{x} = \mathbf{p}(r). \end{aligned} \tag{1}$$

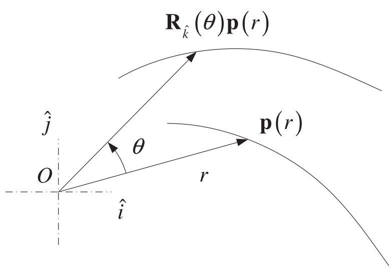

Fig. 1. Parametric definition of a gear profile.