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Compression and torsion of ceramic specimens by application of movable cellular automata

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ABSTRACT

In this paper numerical investigations of the ceramics behaviour have been conducted under quasi-static various loading conditions by application of the Movable Cellular Automata (MCA) method. The paper presents results of simulations of ceramic specimens with and without pores under compression, torsion and torsion of compressed specimens. Weibull analysis has been performed and values of Weibull modulus for all kind of specimens and mechanical loading are given.

1. Introduction

Ceramics are widely used as structural materials due to their many advantages, such as high strength, chemical stability and low density. Their main disadvantage is low fracture toughness compared to metallic materials. The majority of ceramic strength and fracture investigations take place under uniaxial mechanical loading, but in actual use ceramic components are subjected to multi-axial or multi-mode loading. A lot of studies have been devoted to the multi-mode loading applied to quasi-brittle materials, such as rock or concrete, but not monolithic ceramics.

Since relatively little is known about the mechanical behaviour of ceramics in comparison to metallic materials, monolithic ceramics are the subject of interest of materials research. The fracture strength of ceramic specimens under combined modes of mechanical loading has been examined in a few papers only. Experiments with ceramic SiC tubes under combined tension/torsion are presented in [1]. In [2] a generally applied failure criterion is developed for the elements of brittle material under combined tension/torsion based on the normal stress fracture criterion and compare with results of experiments on notched and smooth Al_2O_3 tubes. Combined compression/torsion described in [3] is applied to as-machined and thermal treated specimens. Brittle fracture in graphite specimens with U- and V-notches was investigated experimentally and theoretically under torsion and the results are presented in [4].

In this paper the Movable Cellular Automata (MCA) method introduced in [5] is used to explore defects growth and fracture of elastic-brittle materials. This method allows modelling processes concerning mesoscopic length scale [6,7], such as mass mixing in heterogeneous materials, friction and wear of real systems, cracks generation and development under various modes of mechanical loading [8].

2. The movable cellular automata method

The main foundations of the version of movable cellular automata method applied in this paper have been presented in detail in [9]. Modelled ceramic specimen is divided into elements called cellular automata interacting with each other. One automaton represents particle or grain of material. Automata are placed in three-dimensional space according to the hexagonal system, which provides the highest density of cells. Existing chemical bond between two cells is expressed by linked state between these cells. Movements of a cell depend on its specified physical properties, such as mass, size, shape, and "mechanical properties" represented by coefficients E_p, E_s, E_n. These coefficients express mechanical strength of cell *ijk* to retard and oppose motion in the normal direction to the contact plane between two cells (compressing or stretching) and motion in the tangential direction (rotation). They are evaluated on the base of mechanical properties of chosen material and number of automata comprised in specimen.

In the movable cellular automata method displacement from initial position constitutes local forces of interaction. Force F_n affecting cell *ijk* becomes active if distance in normal direction $r_{n_{ijkmno}}$ to the contact plane between two considered linked automata *ijk* and *mno* increases:

$$F_{n_{ijkmno}} = E_{n_{ijk}} \Delta r_{n_{ijkmno}}. \quad (1)$$

Force Fp appears if automata compress themselves:

$$F_{p_{ijkmno}} = E_{p_{ijk}} (2\rho - r_{ijkmno}), \quad (2)$$

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Table 1
Mechanical properties of ceramics and parameters of ceramic specimens.

Young's modulus (GPa)	390
Density (g/cm^3)	3.01
Compressive strength (GPa)	1.5
Torsional strength (MPa)	100
High (mm)	80
Cross-sectional area (mm^2)	77.78

Table 2
Results of numerical simulations for compression and torsion of specimens without pores.

Specimen	Moment of inertia (kg/m^2)	Compression: σ (MPa)	Torsion: angle (rad)
1	2.072646×10^{-7}	1062.87	0.0153065
2	2.070934×10^{-7}	1058.93	0.0148702
3	2.070709×10^{-7}	1057.56	0.0149749
4	2.074012×10^{-7}	1057.80	0.0148702
5	2.070402×10^{-7}	1063.04	0.0152716
6	2.072838×10^{-7}	1069.55	0.0151669
7	2.072534×10^{-7}	1066.13	0.0151769
8	2.070956×10^{-7}	1068.03	0.015312
9	2.072736×10^{-7}	1070.16	0.015324
10	2.071745×10^{-7}	1066.20	0.0151146
11	2.071280×10^{-7}	1068.86	0.0151639
12	2.071402×10^{-7}	1070.06	0.0150273

Table 3
Results of numerical simulations for torsion of compressed specimens without pores.

Specimen	Angle (rad) for σ = 21.7 (MPa)	Angle (rad) for σ = 43.4 (MPa)
1	0.0121999	0.00863938
2	0.0119904	0.00863938
3	0.0118682	0.00877901
4	0.0118159	0.00816814
5	0.011781	0.0087441
6	0.012322	0.00876155
7	0.0121475	0.00863938
8	0.0121126	0.00863938
9	0.0124267	0.00837758
10	0.0122173	0.00867429
11	0.0122522	0.00870919
12	0.0120951	0.00867429

Table 4
Results of numerical simulations for compression and torsion of specimens with 2% of pores.

Specimen	Moment of inertia (kg/m^2)	Compression: σ (MPa)	Torsion: angle (rad)
1	2.039350×10^{-7}	936.123	0.0148178
2	2.037655×10^{-7}	936.826	0.0146608
3	2.038326×10^{-7}	942.001	0.0143466
4	2.041340×10^{-7}	941.252	0.0147306
5	2.037340×10^{-7}	935.054	0.0145211
6	2.040560×10^{-7}	941.69	0.014591
7	2.039373×10^{-7}	934.565	0.0145386
8	2.038701×10^{-7}	927.276	0.0145735
9	2.040228×10^{-7}	942.756	0.0149749
10	2.039761×10^{-7}	923.796	0.0145735
11	2.038654×10^{-7}	934.232	0.0149226
12	2.039112×10^{-7}	943.133	0.0150447

where r_{ijkmno} is distance between cells and ρ is radius of cells.

Force F_s becomes active if displacement $r_{s_{ijkmno}}$ occurs in the tangential direction to the contact plane:

$$F_{s_{ijkmno}} = E_{s_{ijk}} \Delta r_{s_{ijkmno}}. \quad (3)$$

If cells are in contact, forces of dry and viscous friction are determined. Dry friction force F_d formulas differ if both cells stay in immobility in the direction of contact's plane or if they move, then the force counteracts movements. Precise formulas are following:

$$F_{d_{ijkmno}} = \begin{cases} f_{s_{ijkmno}}, & \text{if } |f_{s_{ijkmno}}| < \mu |f_{n_{ijkmno}}| \\ \mu |f_{n_{ijkmno}}|, & \text{if } |f_{s_{ijkmno}}| \geqslant \mu |f_{n_{ijkmno}}| \end{cases}, \quad (4)$$

where μ is coefficient of dry friction, f_s is tangential and f_n is normal component of impact forces affecting automaton $\vec{f}_{ijk} = \sum_{mno}(\vec{F}_{n_{ijkmno}} + \vec{F}_{s_{ijkmno}} + \vec{F}_{p_{ijkmno}})$.

Viscous friction force F_v is proportional to the relative velocity $\vec{v}_{ijkmno} = \vec{v}_{ijk} - \vec{v}_{mno}$ and it is expressed by equation:

$$F_{v_{ijkmno}} = \eta v_{ijkmno}, \quad (5)$$

where η is the coefficient of viscous friction.

Table 5
Results of numerical simulations for torsion of compressed specimens with 2% of pores.

Specimen	Angle (rad) for σ = 21.7 (MPa)	Angle (rad) for σ = 43.4 (MPa)
1	0.0115541	0.00748746
2	0.0116413	0.00679806
3	0.0113272	0.0066497
4	0.0114319	0.00721694
5	0.0116937	0.0066497
6	0.011781	0.0071384
7	0.0119381	0.00731293
8	0.0114668	0.00714712
9	0.0118682	0.00748746
10	0.011589	0.00733038
11	0.0113185	0.00688532
12	0.0116064	0.00701622

Table 6
Results of numerical simulations for compression and torsion of specimens with 10% of pores.

Specimen	Moment of inertia (kg/m^2)	Compression: σ (MPa)	Torsion: angle (rad)
1	1.910395×10^{-7}	706.458	0.0135263
2	1.910397×10^{-7}	722.288	0.0126885
3	1.907162×10^{-7}	715.345	0.0132994
4	1.911268×10^{-7}	728.948	0.013247
5	1.909232×10^{-7}	724.545	0.0132296
6	1.911194×10^{-7}	719.4	0.0135787
7	1.910423×10^{-7}	722.695	0.0137183
8	1.908177×10^{-7}	712.886	0.0135612
9	1.911585×10^{-7}	717.259	0.0128282
10	1.909137×10^{-7}	730.076	0.0128456
11	1.909252×10^{-7}	736.151	0.0124093
12	1.909634×10^{-7}	720.15	0.0130376

Table 7
Results of numerical simulations for torsion of compressed specimens with 10% of pores.

Specimen	Angle (rad) for σ = 21.7 (MPa)	Angle (rad) for σ = 43.4 (MPa)
1	0.00909317	0.00010472
2	0.00804597	0.000261799
3	0.0092677	0.000471239
4	0.00890118	0.00164061
5	0.00877901	0.00015708
6	0.00890118	0.000401426
7	0.00862193	0.0000698132
8	0.00851721	0.000314159
9	0.00904081	0.000471239
10	0.00932006	0.00139626
11	0.00879646	0.000401426
12	0.00916298	0.000663225