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A damage-mechanics model for fracture nucleation and propagation

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ABSTRACT

In this paper, a composite model for earthquake rupture initiation and propagation is proposed. The model includes aspects of damage mechanics, fiber-bundle models, and slider-block models. An array of elements is introduced in analogy to the fibers of a fiber bundle. Time to failure for each element is specified from a Poisson distribution. The hazard rate is assumed to have a power-law dependence on stress. When an element fails it is removed, the stress on a failed element is redistributed uniformly to a specified number of neighboring elements in a given range of interaction. Damage is defined to be the fraction of elements that have failed. Time to failure and modes of rupture propagation are determined as a function of the hazard-rate exponent and the range of interaction.

1. Introduction

A composite model is introduced in this paper for the nucleation and propagation of fractures. The model incorporates aspects of damage mechanics, fiber-bundle models, and slider-block models. A square array of elements is considered, these elements are analogous to the fibers in a fiber-bundle model and the blocks in a slider-block model. At time $t = 0$ a constant force is applied to the system. Time-to-failure statistics are prescribed. When an element fails the stress on that element is transferred to a prescribed range of adjacent elements. Numerical simulations are used to study the conditions under which a well defined rupture nucleates and to illustrate the propagation of this fracture over the array.

The model is closely related to the fiber-bundle model. The fiber bundle initially consists of n_0 fibers. Subsequently n_f fibers fail and when $n_f = n_0$ the bundle fails. When a fiber fails the load on that fiber is transferred to other fibers. In the equal-load sharing case the load is transferred to all other fibers equally. In the local load sharing case the load is transferred to the adjacent fibers within a prescribed interaction region. Two failure criteria have been proposed. The first is static and a failure strength is prescribed statistically for each fiber [1]. As the stress on the fibers increase, more fibers fail. The second failure criterion specifies a statistical time to failure for each fiber that is stress dependent [2,3]. In terms of applicability the latter approach is now generally accepted. A general review of fiber-bundle models has been given in [4].

Damage mechanics is an empirical continuum approach to material failure [5,6]. A continuum damage variable α is defined by the relation

$$E = E_0(1 - \alpha) \tag{1}$$

where E is the Young's modulus for the damaged material and E_0 is the Young's modulus for the undamaged material. When $\alpha = 1$ failure occurs. A rate equation for the increase in damage is specified. There is a close association between the equal-load sharing fiber-bundle model and damage mechanics if it is assumed that [7]

$$\alpha = \frac{n_f}{n_0} \tag{2}$$

Damage mechanics does not consider the propagation of a rupture.

Slider-block models have been studied extensively as models for earthquakes [8–12]. An array of slider blocks is pulled along a surface with puller springs. When the stress on a block exceeds the static coefficient of friction it slips and stress is transferred to other blocks by connector springs. Extensive studies of the role of stress transfer have been carried out [13]. When a block fails the stress on the block is redistributed equally to neighboring blocks in a given range of interaction. This approach has been used to study a slider-block model in a failure mode [14]. However, no time to failure statistics were incorporated.

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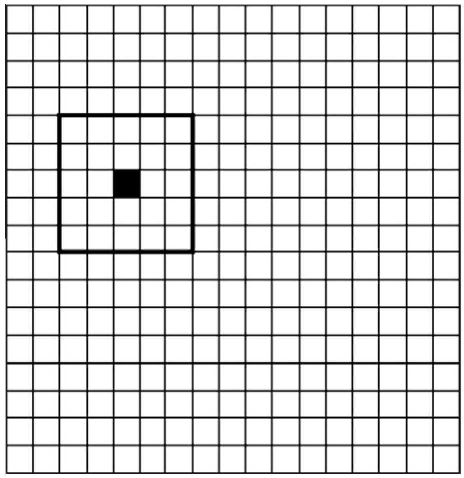

Fig. 1. Illustration of the model. A square grid of elements with width L is considered, in this case $L = 17$ so that there are $n_0 = 17^2 = 289$ elements. At time $t = 0$ a uniform load F_0 is uniformly distributed to all the elements. The stress σ_0 on each element is $\sigma_0 = F_0/n_0$. One element has the shortest time to failure $t_{f,\min}$, the failure of this element is the solid square. The stress from this failed element is redistributed equally to all the elements within range r. Here $r = 2$ and is represented by the black line surrounding the failed element. The stress is redistributed equally to the 24 remaining elements within the square.

2. The model

An illustration of the model is given in Fig. 1. A square grid of elements is considered with L elements on a side, for the case illustrated $L = 17$ so that the total number of elements is $n_0 = 17^2 = 289$. At time $t = 0$ a constant load F_0 is applied to the grid and it is uniformly distributed so that each element has a stress

$$\sigma_0 = \frac{F_0}{n_0} \tag{3}$$

No additional external forces are applied to the system. A time to failure t_f is assigned randomly to each element from a prescribed distribution. In this paper, it is assumed that this distribution is Poisson so that there is no memory of the stress history of an element. The cumulative history of failure times is thus given by

$$P_c(t_f) = 1 - e^{-\nu t_f} \tag{4}$$

where ν is the hazard rate. This distribution has been shown to be applicable to the distribution of nucleation times in solid-state physics, specifically the Ising model [15]. A second assumption is that the hazard rate $\nu(\sigma)$ has a power-law dependence on the stress σ on the element

$$\nu(\sigma) = \nu_0\left(\frac{\sigma}{\sigma_0}\right)^{\rho} \tag{5}$$

where the power-law exponent ρ must be specified and ν_0 is the value of the hazard rate when $\sigma = \sigma_0$. It is found experimentally that values of ρ are in the range 2–5 for various fibrous materials [16].

Initially, at $t = 0$, the stresses on all elements are equal with the value σ_0 given in Eq. (3). For each element a random number P_c in the range 0–1 is chosen. Using this random number the corresponding failure time of the element is obtained from Eq. (4) with $\nu = \nu_0$

$$t_f = \frac{1}{\nu_0}\ln[(1 - P_c)^{-1}] \tag{6}$$

The first element fails at $t = t_{f,min}$ the smallest of these failure times. This first failed element is illustrated in Fig. 1. The failed element is removed from the grid (there is no healing). The stress on the failed element is redistributed equally to all surviving elements in a range of interaction r. For the example illustrated in Fig. 1 the range of interaction is $r = 2$. The redistribution is carried out over a square region with $2r + 1$ elements on a side. The maximum number of elements n_{rd} over which the stress is redistributed is

$$n_{rd} = (2r + 1)^2 - 1 \tag{7}$$

For the example in Fig. 1, $n_{rd} = 24$. In subsequent redistributions some of the elements in the region may have been removed due to previous failures. In this case the stress is redistributed equally to the surviving elements. If the failed element has no surviving neighbors, the stress on that element is dissipated from the system, reducing the total load.

All surviving elements in the grid are given a new time to failure Δt_f that is determined from Eqs. (4) and (5) written in the form

$$\Delta t_f = \frac{1}{\nu_0}\left(\frac{\sigma_0}{\sigma}\right)^{\rho}\ln[(1 - P_c)^{-1}] \tag{8}$$

where P_c is a new random number in the range 0–1. This approach is appropriate for the Poisson distribution of failures given by Eq. (4) since a surviving element has no memory of the prior stress history. Considering the values Δt_f for all elements, the shortest time to failure is determined. At this time this failed element is removed from the grid. This process is continued until all elements have failed. This is the failure time t_{gf} for the grid. At this time the number of failed elements n_f is equal to the number of elements originally on the grid n_0, $n_f = n_0$. Following the standard association of damage mechanics with the fiber-bundle model we take the damage variable α to be given by Eq. (2). The damage variable is the fraction of failed elements, failure of the grid occurs at $\alpha = 1$. A primary object of our simulations is to determine α as a function of t ($0 \leqslant t \leqslant t_{gf}$).

3. Mean-field analysis

The case in which the stress on a failed element is redistributed equally to the surviving elements on the grid can be solved analytically. This is known as equal-load sharing and is the mean-field limit for this problem. For this case the stresses on all surviving elements $\sigma_{mf}(t)$ are equal. The condition that the total force F_0 on the grid remains constant for $t > 0$ requires

$$(n_0 - n_f)\sigma_{mf} = n_0\sigma_0 \tag{9}$$

The standard breakdown rule for the rate of failure of fibers given in [2–4] is

$$\frac{d(n_0 - n_f)}{dt} = -\nu(\sigma)(n_0 - n_f) \tag{10}$$

where ν is again the hazard rate. If ν is a constant ν_0 the integration of Eq. (10) gives the probability distribution given in Eq. (4). It is important to note that there is a correspondence between the fiber-bundle formulation and the damage formulation only if the Poissonian failure condition in Eq. (4) is used.

The power-law dependence of the hazard rate on stress as given in Eq. (5) is assumed to be valid. Combining Eqs. (5) and (9) gives

$$\nu(n_f) = \nu_0\left(1 - \frac{n_f}{n_0}\right)^{-\rho} \tag{11}$$

and substitution into Eq. (10) gives

$$\frac{d(n_0 - n_f)}{dt} = \frac{-\nu_0 n_0^{\rho}}{(n_0 - n_f)^{\rho-1}} \tag{12}$$

Integrating with $n_f = 0$ at $t = 0$ with the definition of the damage variable α given in Eq. (2) gives