Mechanism and Machine Theory 45 (2010) 1568–1582

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Prediction of mechanical gear mesh efficiency of hypoid gear pairs

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ARTICLE INFO

Article history:
Received 25 December 2009
Received in revised form 22 June 2010
Accepted 23 June 2010
Available online 24 July 2010

Keywords:
Hypoid gears
Gear efficiency
Friction coefficient
Loaded tooth contact analysis

ABSTRACT

This study proposes a new spiral bevel and hypoid gear mechanical efficiency model for both face-milling and face-hobbing type cutting methods. The proposed efficiency model combines a computationally efficient contact model and a mixed elastohydrodynamic lubrication (EHL) based surface traction model to predict friction power losses. The employed contact model simulates the cutting process to compute all required geometric parameters of the contacting surfaces. It computes the unloaded contact positions between the tooth surfaces utilizing an ease-off approach and estimates the tooth compliance using a shell model. It also computes pressure distribution and rolling and sliding velocities. The EHL traction model considers typical ranges of the key contact parameters governed by hypoid gear applications, including Hertzian pressure, contact radii, surface speeds, lubricant temperature and surface roughness amplitude of hypoid type of gears, covering a wide range of conditions from full-film to boundary lubrication regime. At the end, the efficiency model is applied to two face-hobbed examples with similar overall dimensions, but different shaft offsets to investigate the influences of key operating and design parameters on the mechanical gear mesh power losses.

1. Introduction

Prediction of power losses of automotive drive trains is becoming an increasingly critical task for power train designers. This is mainly because the government regulations in regards to fuel economy and carbon emissions are becoming more stringent. Forecasted increases in oil prices also add to the motivation to predict and reduce power losses of drive trains. In rear-wheel drive vehicles, the rear axle-differential unit is one of the major sources of power losses. The axle efficiency values can be typically as low as 90 to 95% [1,2]. Considering that rear-wheel or all-wheel drive vehicles comprise a significant share of the global passenger vehicle market, any sizable improvements to the axle efficiency can have a significant positive impact on the environment and the energy consumption.

Axle power losses can be divided into two groups. One group of losses is independent of the torque transmitted. These load independent (spin) power losses are due to viscous bearing losses (including the losses due to pre-load) and gear windage and/or oil churning losses, which have been studied in several recent studies for spur gear pairs [3–5]. While such losses are also relevant to cross-axis gearing, they are outside the scope of this research. The other groups of losses are induced by friction at the bearing and gear pair contacts under load. These load-dependent (mechanical) power losses of hypoid gear pairs are especially significant due to excessive relative sliding experienced by the lubricated gear tooth contacts. The shaft offset, being the main difference between spiral bevel and hypoid gears increases the relative sliding further, causing higher levels of friction induced power losses [6]. The motivation behind this study is to develop a mathematical model to predict the mechanical power losses of face-milled and face-hobbed hypoid gears.

Modeling mechanical losses of a gear pair involves (i) computation of surface geometry parameters and velocities, and the normal load at each contact point from a tooth contact analysis model, (ii) computation of the coefficient of friction at each contact

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doi:10.1016/j.mechmachtheory.2010.06.015

point using a lubrication-based friction model, (iii) computation of the surface traction from the distributions of the friction coefficient and normal force, and (iv) computation of the friction torque and the resultant power loss. Published gear efficiency models differ mostly in the way they determine the friction coefficient as well as the way the loaded tooth contact analysis is performed. In terms of the friction models, one group of earlier studies used a constant user-defined friction coefficient μ [7–9] in computing the power losses. Recognizing the fact that μ is dependent on various contact parameters, including rolling velocity, slide-to-roll ratio, radii of curvature of the contacting surfaces and the normal load, all of which vary as gears rotate in mesh, various empirical μ formulae [10–13] developed for different types of contacts and lubricants were employed by a second group of studies [14–18]. However, the applicability of these models was limited to narrow ranges of the operating temperatures, speed, load, and surface roughness conditions represented by the empirical formula. The third group of models predicted the friction coefficient or direct power losses at the gear contact interfaces by using elastohydrodynamic lubrication (EHL) models of varying complexity [19–23]. This approach, while physics-based and potentially more accurate, requires a significant computational effort as several hundreds of EHL analyses are typically required to predict the mechanical losses of a gear pair. In order to avoid this difficulty, Xu et al. [24] proposed a methodology to derive a gear contact friction formula up-front by using the EHL model of Cioc et al. [25]. Using this EHL model, they conducted a large parameter study, covering wide ranges of contact and surface parameters as well as operating conditions representative of gears. The predicted surface traction data was reduced into a single formula by using linear regression technique. The EHL model used in Xu et al. could only include limited asperity contacts. Li et al. [26] used the same regression based approach later with a truly mixed EHL model [23,26–28] to capture the roughness effects more accurately.

All of the models cited above were limited to spur or helical gears. Efficiency models for hypoid gears are very sparse. Approximating the hypoid gear power loss as the sum of losses from the corresponding spiral bevel and worm gears, Buckingham [29] recommended a power loss equation. Coleman [1] proposed a simple closed-form formula to estimate bevel and hypoid gears efficiency. This heuristic formula used a constant friction coefficient of $\mu = 0.05$ at every contact point and was a function of the normal load, pressure angle, and pinion and gear mean spiral angles. Simon [30,31] applied a smooth surface EHL model to simulate hypoid gear lubrication. A model proposed recently by Xu and Kahraman [32] extended their helical gear efficiency model to hypoid gears. They used a commercially available FE-based hypoid gear contact model CALYX [33] to determine all required contact load and geometry parameters including curvatures. Employing set of equations developed by Litvin [34] primarily to describe relationships between curvatures of mating surfaces, they computed sliding and rolling velocities at each contact point along and perpendicular to the contact line. While this model [32] was physics-based and included most of the key surface, lubricant, geometry and operating parameters, its FE load distribution computation required significant computational time, making it impractical for design and parameter sensitivity studies. It relied on the same FE model for its geometry and curvature information as well. More importantly, its EHL model [25] to derive the friction coefficient formula was not designed for simulating mixed and boundary lubrication conditions. Therefore, the fidelity of the model of Xu and Kahraman [32] proposed was limited to contact conditions with very limited asperity interactions. However, in most automotive hypoid gear applications, mixed or boundary EHL conditions characterized by excessive metal-to-metal contacts of the asperity peaks occur commonly, warranting the use of a mixed EHL model such as the one proposed by Li and Kahraman [26,27] to handle any lubricated gear contact conditions ranging from almost dry to full-film EHL.

2. Hypoid gear mechanical power loss model

The hypoid gear mechanical efficiency model proposed in this study improves the methodology of Xu and Kahraman [32] by (i) deriving the contact curvature and surface velocity values at each contact point directly from the surface geometries instead of relying on any particular commercial FE package, (ii) employing the ease-off based loaded tooth contact model of Kolivand and Kahraman [35] for computation of the normal load for minimizing the computational time required for this task, and (iii) by incorporating a new μ formula obtained by using the mixed EHL model of Li and Kahraman [26] for line contacts such that any degree of asperity interactions can be modeled. Fig. 1 shows the flowchart of the methodology used to compute the mechanical power loss of a hypoid gear pair, combining these two models [26,35].

Overall methodology of the hypoid gear loaded tooth contact model is shown in flowchart of Fig. 2. Blank dimensions, machine settings, cutter geometry, misalignment, load and speed are input data for the hypoid gear contact model, which usually are in form of a design file of data provided by hypoid gear design packages to hypoid gear manufacturers. The contact model, as described in detail in Ref. [35] contains the several sub-models. First of all, a cutting simulation model applies the fundamental equation of meshing between cutter and corresponding blank for both face-milling and face-hobbing methods to compute surface coordinates, normal vectors and principal curvatures and directions of both pinion and gear. The input for this model includes machine settings, the cutter geometry parameters and the blank dimensions. Next, an unloaded tooth contact model constructs and uses the ease-off topography and surface of roll angle (with the misalignment applied) to compute potential contact lines, and equivalent radius of curvatures. A semi-analytical compliance model based on Rayleigh–Ritz shell formulation is used to compute tooth compliances. Closed-form Weber [36] equation is also used for computing the Hertzian deformations. Base rotation effects are introduced approximately using a method similar to the one proposed by Stegemiller and Houser [37] for helical gears. Finally, a loaded tooth contact analysis model solves the compatibility and equilibrium equations simultaneously to compute load distribution. We refer to Ref. [35] for details of this model and focus here on the definition of the parameters required by the friction model according to the flowchart of Fig. 1.

At a given time step k with pinion roll angle $q = q^k = k\Delta q$ ($k \in [1, n_s]$), there are n_{cl} number of potential contact lines between the gear and pinion surfaces of adjacent tooth pairs (depending on the contact ratio and the pinion roll angle). Here, n_s is total