Available online at www.sciencedirect.com
ScienceDirect
Mechanism and Machine Theory 44 (2009) 272–288
Mechanism and Machine Theory
www.elsevier.com/locate/mechmt

# Free vibrations analysis of a rotating shaft with nonlinearities in curvature and inertia

S.A.A. Hosseini, S.E. Khadem *

*Department of Mechanical Engineering, Tarbiat Modarres University, P.O. Box 14115-177, Tehran, Iran*

Received 23 April 2007; received in revised form 4 December 2007; accepted 21 January 2008
Available online 14 March 2008

**Abstract**

In this paper the free vibrations of an in-extensional simply supported rotating shaft with nonlinear curvature and inertia are considered. Rotary inertia and gyroscopic effects are included, but shear deformation is neglected. To analyze the free vibrations of the shaft, the method of multiple scales is used. This method is applied to the discretized equations, and directly to the partial differential equations of motion, which demonstrates the same results. An expression is derived which describes the nonlinear free vibrations of the rotating shaft in two transverse planes. It is found that in this case, both forward and backward nonlinear natural frequencies are being excited. The results of perturbation method are validated with numerical simulations.


*Keywords:* Rotating shaft; Free vibrations analysis; Large amplitude vibrations; Nonlinear curvature and inertia; Multiple scales method

## 1. Introduction

Rotating shafts are used for power transmission in many modern machines. Accurate prediction of dynamics of rotating shafts is necessary for a successful design. Free vibrations analysis is one of the important steps in rotor-dynamics. Grybos [1] considered the effect of shear deformation and rotary inertia of a rotor on its critical speeds. Choi et al. [2] presented the consistent derivation of a set of governing differential equations describing the flexural and the torsional vibrations of a rotating shaft where a constant compressive axial load was acted on it. Jei and Leh [3] investigated the whirl speeds and mode shapes of a uniform asymmetrical Rayleigh shaft with asymmetrical rigid disks and isotropic bearings. Free damped flexural vibrations analysis of composite cylindrical tubes was carried out by Singh and Gupta [4], where they used beam and shell theories. Sturla and Argento [5] studied the free and forced response of a viscoelastic spinning Rayleigh shaft. Melanson and Zu [6] studied the free vibrations and stability of internally damped rotating shafts with general boundary conditions. Kim et al. [7] studied the free vibrations of a rotating tapered composite Timoshenko shaft.

* Corresponding author. Tel./fax: +98 21 88011001 3388.
*E-mail address:* Khadem@Modares.ac.ir (S.E. Khadem).

0094-114X/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mechmachtheory.2008.01.007

Karunendiran and Zu [8] analyzed free vibrations analysis of a shaft on resilient bearings. Free and forced vibrations analysis of a rotating disk-shaft system with linear elastic bearings was investigated by Shabaneh and Zu [9]. Bearings were mounted on viscoelastic suspensions. El-Mahdy and Gadelrab [10] studied the free vibrations of unidirectional fiber reinforcement composite rotor. Raffa and Vatta [11] derived the equations of motion for an asymmetric Timoshenko shaft with unequal principal moments of inertia. The critical speeds and mode shapes of a spinning Rayleigh beam with six general boundary conditions are investigated analytically by Sheu and Yang [12]. Gubran and Gupta [13] studied the effect of stacking sequence and coupling mechanisms on the natural frequencies of composite shafts.

To simplify the analysis, researchers often try to use the linear analysis. But, application of nonlinear analysis is sometimes inevitable. Many phenomena should be described with nonlinear equations which are not explainable with linear analysis. Using the Hopf bifurcation theory, Kurnik [14] analyzed self-excited vibrations of a rotating geometrically nonlinear shaft caused by internal friction. Shaw and Shaw [15] analyzed stability and bifurcations of a rotating shaft made of a viscoelastic material. Using the general theory of bar bending, Leinonen [16] presented a nonlinear model to describe the bending behavior of a rotating shaft. Kurnik [17] analyzed the stability and self-excited postcritical whirling of a rotating shaft with the aid of bifurcation theory. The shaft was made of a material with elastic and viscous nonlinearities. Vibrations of a spinning rotor with nonlinear elastic and geometric properties were considered by Cveticanin [18]. The method of multiple scales was applied to analyze the free and forced vibration of nonlinear rotor-bearing systems by Ji and Zu [19]. They used a nonlinear spring and linear damping to model the nonlinear bearing pedestal. A geometrically nonlinear model of a rotating shaft was introduced by Luczko [20]. The model included Von-Karman nonlinearity, nonlinear curvature effects, large displacements and rotations as well as gyroscopic and shear effects. Viana Serra Villa et al. [21] used the invariant manifold approach to explore the dynamics of a nonlinear rotor. They constructed a reduced order model with the aid of nonlinear normal modes and evaluated its performance. Cveticanin [22] considered the free vibrations of a Jeffcott rotor with cubic nonlinear elastic property. He applied the Krylov–Bogolubov method to solve the nonlinear equations of motion. Lately, present authors studied free vibrations of a rotating beam with random properties [23], and vibrations and reliability of a rotating beam with random properties under random excitations [24]. To study uncertainty, stochastic finite element method based on the second order perturbation method was applied.

In this paper, the equations of motion of a continuous simply supported rotating shaft with nonlinear curvature and inertia are derived. Rotary inertia and gyroscopic effects are included but, shear deformation is neglected. Using the in-extensionality assumption, the equations of motion are derived with the aid of Hamilton principal. To solve theses gyroscopic nonlinear equations of motion approximately, the multiple scales method is used. This method is applied directly to the partial differential equation of motion and to the discretized equations. Some researches have shown that applying the multiple scales method to the discretized equations might produce quantitative and qualitative errors (for example, [25]). The systems that they considered were nongyroscopic. Here, it is shown that in our gyroscopic system, resulting reduced equations from two approaches are the same. An expression is derived which describes the nonlinear free vibration of the rotating shaft in two transverse planes. Some authors have used only forward whirling frequency to study the nonlinear free vibrations of a rotating shaft with gyroscopic effects (for example, [19]). Here, it is shown that in the nonlinear free vibrations of a rotating shaft with gyroscopic effects, both forward and backward nonlinear natural frequencies are excited. So, if one takes into account only the forward natural frequencies, the results become incorrect. Effects of rotary inertia, external damping coefficient and rotating speed on the nonlinear amplitude and natural frequencies of first two modes of a shaft are examined. The results of perturbation method are validated with numerical simulations.

## 2. Equations of motion

The schematic of a continuous rotating shaft has been shown in Fig. 1. The length of the undeformed shaft center line is $l$. Displacements of any particle of the shaft are described in inertial frame $X$–$Y$–$Z$. The $x$–$y$–$z$ constitute a local coordinate which are principal axes of the beam cross section. The axes are attached to the center line of the deformed shaft (Fig. 1) at position $x$. Displacements of a particle in arbitrary location $x$ along $X$-, $Y$- and $Z$-axes are $u(x,t)$, $v(x,t)$ and $w(x,t)$, respectively, and torsional angle is $\phi(x,t)$. Following