Applied Mathematical Modelling 63 (2018) 148–171

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Three-dimensional complex variable element-free Galerkin method

Xiaolin Li*

[a] School of Mathematical Sciences, Chongqing Normal University, Chongqing 400047, PR China
[b] Key Laboratory for Optimization and Control Ministry of Education, Chongqing Normal University, Chongqing 400047, PR China

ARTICLE INFO

Article history:
Received 30 December 2017
Revised 26 May 2018
Accepted 25 June 2018

Keywords:
Meshless methods
Complex variable moving least squares approximation
Complex variable element-free Galerkin method
Error estimation
Three-dimensional problem
Wave equation

ABSTRACT

The complex variable element-free Galerkin (CVEFG) method is an efficient meshless Galerkin method that uses the complex variable moving least squares (CVMLS) approximation to form shape functions. In the past, applications of the CVMLS approximation and the CVEFG method are confined to 2D problems. This paper is devoted to 3D problems. Computational formulas and theoretical analysis of the CVMLS approximation on 3D domains are developed. The approximation of a 3D function is formed with 2D basis functions. Compared with the moving least squares approximation, the CVMLS approximation involves fewer coefficients and thus consumes less computing times. Formulations and error analysis of the CVEFG method to 3D elliptic problems and 3D wave equations are provided. Numerical examples are given to verify the convergence and accuracy of the method. Numerical results reveal that the CVEFG method has better accuracy and higher computational efficiency than other methods such as the element-free Galerkin method.

1. Introduction

Meshless (or meshfree) methods have been developed and achieved prominent progress in the past 25 years for the numerical solution of science and engineering problems. Compared with mesh-based methods, the formulation of shape functions in meshless methods requires only nodes with no mesh. The moving least squares (MLS) approximation [1] is an efficient and frequently used technique to form meshless shape functions. Many meshless methods, such as the element-free Galerkin (EFG) method [2], the meshless local Petrov–Galerkin (MLPG) method [3], the boundary node method (BNM) [4] and the symmetric Galerkin BNM [5] have been developed using the MLS approximation. Besides, theoretical results and improvements of the MLS approximation have also been established [6–8]. The MLS approximation can form shape functions with high accuracy and smoothness. However, since the MLS approximation of an n-dimensional function is formed exactly with n-dimensional basis functions, many coefficients are involved in the approximation and thus lots of nodes are required in the MLS-based meshless methods [7].

To reduce the number of coefficients in the MLS approximation and hence to improve the computational efficiency of the MLS-based meshless methods, Cheng et al. proposed the complex variable moving least squares (CVMLS) approximation by introducing the complex variable theory into the MLS approximation [9,10]. By expressing a 2D point $\mathbf{x} = (x_1, x_2)^{\mathrm{T}}$ as $z = x_1 + \mathrm{i}x_2$ with one variable z, the CVMLS approximation of a 2D function is formed with 1D basis functions. Then, fewer

* Corresponding author at: School of Mathematical Sciences, Chongqing Normal University, Chongqing 400047, PR China.
E-mail address: lxlmath@163.com

https://doi.org/10.1016/j.apm.2018.06.040

coefficients and fewer nodes are required in constructing shape functions. Some meshless methods, such as the complex variable element-free Galerkin (CVEFG) method [11], the complex variable meshless local Petrov–Galerkin method [12], the complex variable boundary element-free method [13] and the complex variable Galerkin boundary node method (CVGBNM) [14] have been developed based on the CVMLS approximation. To take the merits of these complex variable meshless methods, the complex variable theory has also been introduced into the reproducing kernel particle method [15] and the moving Kriging interpolation [16]. Moreover, Bai et al. proposed the improved CVMLS approximation and the improved CVEFG method by using conjugate basis functions [17,18]. Compared with the original EFG method, these CVEFG methods have higher accuracy and computational efficiency.

The complex variable theory is valid and perfect for 2D space. In previous papers, the CVEFG method and other complex variable meshless methods have been applied to many 2D science and engineering problems [7,9–19]. Very good results are gained in solving these problems. However, these papers only provide applications to 2D problems and there are no indications if their discoveries can be applied to 3D problems. In Refs. [20,21], Cheng et al. stated that the CVEFG method cannot be applied to 3D problems owing to the use of the complex variable theory. This is the main deficiency of the CVEFG method and other complex variable meshless methods.

The solution of problems in 3D domain is generally much more complex than in 2D domain owing to the increase of the computational burden and the troubles in the discretization of the domain. To the best of our knowledge, very few published papers are devoted to the development of complex variable meshless methods for 3D problems. In Refs. [20,21], Cheng et al. proposed a dimension splitting CVEFG method for 3D potential and transient heat conduction problems. In their work, a 3D problem is transformed into a set of 2D ones by using the dimension splitting technique, and then the CVEFG method is used in 2D domain and the finite difference discretization is used in the splitting direction. Besides, Li et al. proposed a CVGBNM for 3D potential, Helmholtz and Stokes problems [22]. With the aid of boundary integral equations [23,24], the CVMLS approximation in the CVGBNM is performed only on the 2D bounding surface of a 3D domain. In these papers, the CVMLS approximation is not used in a 3D space but a 2D space. On the other hand, owing to the use of special techniques, the dimension splitting CVEFG method inherits the shortcomings of the dimension splitting technique, while the CVEFG method inherits the shortcomings of boundary integral equations. In this paper, the CVMLS approximation and the CVEFG method are developed in a generic 3D space.

The first goal of this paper is to develop and analyze a 3D CVMLS approximation. Formulations of the CVMLS approximation on 3D domains are developed in detail. Then, properties, stability and error of the 3D CVMLS approximation are discussed theoretically. By expressing a 3D point $\mathbf{x}=(x_1,x_2,x_3)^{\mathrm{T}}$ as $\mathbf{x}=(z,x_3)^{\mathrm{T}}$ with two variables $z=x_1+\mathrm{i}x_2$ and x_3, the CVMLS approximation of a 3D function is formed with 2D basis functions. Then, the CVMLS approximation involves fewer coefficients and requires fewer nodes than the MLS approximation.

The second goal of this paper is to develop and analyze a CVEFG method for 3D problems. In the CVEFG method, shape functions are formulated by the CVMLS approximation and are used for approximating the Galerkin weak form of boundary value problems. Computational formulas of the CVEFG method for 3D elliptic problems and 3D wave equations are deduced in detail. Error of the CVEFG method is also analyzed theoretically. Numerical examples are given to show the performance of the method. Convergence and comparison researches are conducted to validate the accuracy, convergence and efficiency.

2. The CVMLS approximation for 3D functions

2.1. Notations

Let Ω be a bounded domain in $\mathbb{R}^3$ with a Lipschitz continuous boundary Γ. For any point $\mathbf{x}=(x_1,x_2,x_3)^{\mathrm{T}}\in\Omega$, let $z=x_1+\mathrm{i}x_2$, where $\mathrm{i}=\sqrt{-1}$ is an imaginary number. Then, the point $\mathbf{x}$ can be denoted as $\mathbf{x}=(z,x_3)^{\mathrm{T}}$ with two variables z and x_3. As a result, the CVMLS approximation of 3D functions can be constructed by using 2D basis functions.

Let $\{\mathbf{x}_I\}_{I=1}^N$ be N scattered nodes in Ω. For any $\mathbf{x}\in\Omega$, we use $\wedge(\mathbf{x})\triangleq\{I_1,I_2,\ldots,I_\tau\}\subseteq\{1,2,\ldots,N\}$ to denote the global sequence numbers of nodes whose influence domains cover $\mathbf{x}$. The influence domain of $\mathbf{x}_I$ is

$$\Re_I\triangleq\Re(\mathbf{x}_I)=\{\mathbf{y}\in\Omega:|\mathbf{y}-\mathbf{x}_I|\le h_I\},\tag{1}$$

where h_I is the radius of $\Re_I$. Let

$$h=\max_{1\le I\le N}\min_{1\le J\le N,J\ne I}\left|\mathbf{x}_I-\mathbf{x}_J\right|$$

be the nodal spacing. For theoretical analysis, let the data site $\{\mathbf{x}_I\}_{I=1}^N$ satisfy the quasi-uniform condition,

$$C_1h\le h_I\le C_2\ h,\qquad I=1,2,\ldots,N,\tag{2}$$

where C_1 and C_2 are positive constants independent of h. In the reproducing kernel particle method [25] and the MLS approximation [26,27], a similar quasi-uniform condition has also been used.

Let

$$w_I(\mathbf{x})=\varphi\left(\frac{|\mathbf{x}-\mathbf{x}_I|}{h_I}\right),\qquad I=1,2,\ldots,N,\tag{3}$$