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Vibrations of inhomogeneous piezoelectric bodies in conditions of residual stress–strain state

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ABSTRACT

In the article, we present the general linearized statement of the boundary problem on vibrations of inhomogeneous piezoelectric body under residual stress–strain state. We have derived the weak statement of the problem for the test functions satisfying the essential boundary conditions, formulated the general variational principle for a prestressed piezoelectric body and proposed several options for the potential energy representation. On the basis of the principles proposed, we have formulated and investigated a number of particular boundary problems on steady-state vibrations of inhomogeneous piezoelectric prestressed rods and thin prestressed disk polarized in the direction of thickness. The analysis of the residual stress levels on frequency response function for the bodies considered is provided.

1. Introduction

Modern inhomogeneous piezoelectrics have been extensively used in the micro- and macro-electromechanical systems and smart devices that are employed in many high-tech industries such as aircraft construction, space technology, nuclear energy, medicine and others. One of the most promising types of inhomogeneous piezoelectrics is functionally gradient piezoelectric (FGPE) featuring a continuous change in the electrical and mechanical properties. Unlike the discrete properties of laminate composites, this property does not result in destruction due to layering caused by stress concentrators in border areas. Thus, the functional gradient in material properties determines the structural integrity and strength of the entire product. The up-to-date methods of manufacturing FGPE, such as powder metallurgy, melting and liquid-phase deposition technology, allow to create materials with predetermined characteristics in a wide range of changes, making them available to use in various fields of technology [1,2]. Despite the fact that initially the materials with functional-gradient properties served as heat-resistant coatings in space research, in the later studies a wide range of other potential applications of these materials has been found including construction of fast breeder reactors [3], materials with gradient refractive index for producing digital disks, piezoelectric and thermoelectric devices, medical implants, thermoelectric generators and many other applications.

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Problems on a propagation of elastic and electroelastic waves in isotropic, anisotropic and piezoelectric materials have been studied for a long time; let us mention fundamental works [4–7] and also some papers with applications of these theories [8–10]. Problems of propagation of surface acoustic waves (SAW) are of particular interest [5–7,11]. Many transducers, actuators, filters, delay lines, oscillators, various devices for signal-processing and visualization of acoustic imaging, as well as devices for ultrasonic, acoustic, and other types of non-destructive testing operate on the basis of SAW of Rayleigh, Love, Lamb, B-G [12]. In practice, they often incorporate prestressed elements in the SAW devices in order to improve their performance or to choose the most optimal conditions of functioning. On the other hand, there is often some unaccounted residual stress–strain state (RSSS) as a result of manufacturing operations, mechanical or thermal influences. It is also known that residual stresses (RS) in piezoelectrics may occur due to polarization of the medium; domain reorientation during the polarization causes significant internal stresses which weaken because of ceramics aging [13]. Sometimes RS produced in piezoelectrics exceed 1 GPa. Importantly, the presence of RSSS causes changes in the SAW propagation velocity and frequency offsets that may be used to control the filtration properties, temperature compensation of devices, etc. Since the quantities characterizing RS and electric fields can reach high values, and the external signal or perturbation are usually small, in order to simulate the behavior of these materials adequately taking into account RSSS factors, it is necessary to use the theory of small perturbations superimposed on finite generalized deformation [14–16].

Three-dimensional linearized theory of deformable solid bodies under residual stress state has begun in the beginning of the XX century. Let us mention a number of scientists who laid down the foundations for this theory: Southwell [17], Biezeno and Hencky [18], Biot [19], Neuber [20], Trefftz [21], Green et al. [22]. Their theories were then refined by Novozhilov [23], Truesdell and Noll [24], Tiersten [14], Washizu [25], Zubov [26], Guz [27], Hoger [28], Robertson [29], Kravchishin and Chekurin [30] and others. The basics of the theory of finite deformations can be found in many literature, for instance, in [23,31–33]. The existing RS models may be distinguished by different degrees of relevance. For example, the homogeneous RS model is rather popular [34,35], however, various structures including piezoelectrics contain essentially inhomogeneous RSSS. Notwithstanding a variety of approaches to the modeling of RSSS in bodies, today there is a lack of review articles or monographs concerning this subject. One of the most comprehensive review on RS models for elasticity problems was made by Bazant [36]. Also it is worth noting the work [12] by Kuang containing a general RS model for the electroelasticity problems.

It should be mentioned that the problem of modeling and computational simulation of rods and plates accounting RSSS and coupled fields is scantly discovered in literature. Among the works in this direction let us mention the papers [37,38] devoted to the development of non-classical nonlinear vibration theories for beams in the framework of the Timoshenko model with considering magnetic, electric, thermal and elastic fields, and initial bending state. Also the work [39] is worth noting where the authors formulated finite-element statement based on the thin plates theory in order with the aim of control of vibration regime of functionally-graded plates with integrated piezoelectric sensors and actuators. In the paper [40] the authors investigated dynamical response of functionally-graded plates supported by elastic foundations under impulsive loading with the help of the Galerkin method and the modal superposition method. The models of inhomogeneous electroelasticity firstly were based on the layer-structure models [41]; nevertheless, due to polarization or repolarization, piezoelectric elements obtain more complicated type of inhomogeneity that implies development of new refined theoretical models and research techniques. Let us mention a number of works [42–44] containing the results of investigation of laminated fuzzy fiber-reinforced composite plates and shells integrated with the piezoelectric patches; the authors developed three-dimensional finite element models in order to study the damping characteristics of the considered composite structures.

The development of methods of reconstruction of properties of various structural elements is of great importance for the modern mechanical engineering. In literature the material is often modeled as homogeneous, so that its properties are characterized by a set of physical constants. At the same time, in many applications the hypotheses of homogeneity is irrelevant, and the properties of the object under consideration should be modeled in dependence of coordinates. In particular, this is typical for structures made of inhomogeneous piezo-active materials like FGPE with properties characterized by a number of functions determined from the experiment; a processing of the experimental data requires involving of the apparatus of coefficient inverse problems, and that that is a complicated mathematical problem. The efficiency and accuracy of modern measuring equipment depends on the theoretical models used as the basis of its work; in particular, it depends on constants and variables of the constitutive relations and governing equations of the models describing the behavior of a material of functional element of some device. Some diagnostic devices and sensors operate on the basis of the phenomenon of direct piezoelectric effect when it is assumed that the material behaves in accordance with the classical linear electroelasticity model [13]. The most common functional elements have a form of simple objects (like plates, rods, cylinders, and others) and made of piezoelectric ceramics. Due to multistage processing steps (several temperature transitions, deposition, molding, etc.), in the final product there are always imperfections and deviations from the established norms, in particular, in the form of inhomogeneity in polarization and elastic properties or RSSS. At the same time, the knowledge of the real properties of an inhomogeneous piezoelectric element allows evaluating its functional characteristics and possibility of its usage in a specific device.

One of the most efficient techniques of implementation of nondestructive reconstruction of arbitrary inhomogeneous properties of solid bodies is the acoustical method. The general principles of proper mathematical formulation and solving of inverse problems on identification of various inhomogeneous characteristics of elastic and thermoelastic bodies on the basis of the acoustical method are described in [45–50]. The monograph [51] presents a few theoretical aspects of modeling