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A coupled model for train-track-bridge stochastic analysis with consideration of spatial variation and temporal evolution

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ABSTRACT

Due to random characteristics of system parameters and excitations, the dynamic assessment and prediction for the train-track-bridge interaction systems become rather complex issues needing to be addressed, especially considering the longitudinal inhomogeneity and uncertainty of dynamic properties in physics and correspondingly their temporal evolutions. In this paper, a temporal-spatial coupled model is developed to fully deal with the deterministically/non-deterministically computational and analytical matters in the train-track-bridge interactions with a novelty, where a train-track-bridge interaction model is newly developed by effectively coupling the three-dimensional nonlinear wheel-rail contact model and the finite element theory, moreover, the Monte-Carlo method (MCM) and Karhunen–Loève expansion (KLE) are effectively united to model the random field of track-bridge systems, and a spectral evolution method accompanied by a track irregularity probabilistic model are introduced to select the most representative track irregularity sets and to characterize their random evolutions in temporal dimension. In terms of random vibration analysis, the high-efficiency and effectiveness of this developed model is validated by comparing to a robust method, i.e., MCM. Apart from validations, multi-applications of the temporal-spatial coupled model from aspects of deterministic computation, random vibration, resonant analysis and long-term dynamic prediction, etc., have been fully presented to illustrate the universality of the proposed model.

1. Introduction

Railway bridges, as a kind of infrastructure, are becoming increasingly important in supporting and guiding the train-track systems. Especially in high-speed lines, the proportion of bridges is much higher than common railway lines that are mainly supported by the subgrade layer. To specific lines, even 90% over is occupied by bridges for conservation of land and environment protection. Hence the theoretical methods and applied technologies related to the assessment of dynamic performance of railway systems, when a train passes through the track/bridge structures, have attracted more and more attentions in last two decades.

Comparing to expensively experimental studies in situ or lab, the dynamic simulations actualized by the computer program have become a dominant strategy in most situations. Till now, the dynamic models developed to characterize the train-track-bridge interactions are numerous, but mostly concentrated on vertical vibrations [1–8], obviously, it limits the re-

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Nomenclature

m_r	the rail mass per unit length
A_r	the cross-sectional area of the rail
W_r	the polar moment of inertia of the rail cross-section
L	the length of the beam element
l_r	the distance between two adjacent rail pads
E_r	the Young's modulus of the rail
(I_{ry},I_{rz})	the flexural moment of inertia about the Y-axis and Z-axis of the cross section of the rail respectively
k_{rt}	the torsional rigidity of the rail cross section around the X-axis
m_s	the mass of the slab per unit volume
(h_s,b_s,l_s)	the height, width and length of the slab track element respectively
E_s	the Young's modulus of the slab track
μ_s	the Poisson ratio of the slab track
I_{sz}	the moment of inertia of the track slab around Z-axis
m_p	the mass of the pier
(k_{px},k_{px},k_{px})	the supporting stiffness coefficients of the subgrade in X-, Y- and Z- axis, respectively
(c_{px},c_{px},c_{px})	the supporting damping coefficients of the subgrade in X-, Y- and Z- axis, respectively
$(k_{rp,z},k_{rp,y})$	the vertical and lateral stiffness of the rail pad respectively
$(c_{rp,z},c_{rp,y})$	the vertical and lateral damping of the rail pad respectively
b	the lateral distance between the contact point of rail pad-slab track and the left-side border of the slab along X-axis
$(k_{ca,z},k_{ca,y})$	the vertical and lateral stiffness coefficients of the CAM
(B_r,H_r)	the central lateral and vertical distances between the slab track and the girder
$(k_{p,z},k_{p,y})$	the vertical and lateral stiffness reflecting the properties of the bearing
a_0	the half of the horizontal distance between two contact points
(r_{li},r_{ri})	the rolling radius of the left and right wheelset respectively
Ω	the nominal rolling angular velocity of the wheelset
(I_{wY},I_{wZ})	the moment of inertia of the wheelset around Y- and Z- axis
M_w	the mass of the wheelset
R_i	the radius of the curvature of the rail corresponding to the ith wheelset
ϕ_{wi}	the angle of superelevation corresponding to the center of the ith wheelset
$(\dot{\varphi}_{wi}, \ddot{\varphi}_{wi})$	the first-order and second-order derivatives of ϕ_i
r_0	the nominal rolling radius of the wheelset
$\dot{\lambda}_{wi}$	the first-order derivative of the curvature
$\bar{g}$	the acceleration of gravity
V	the running speed of the vehicle
M_c	the mass of the car body
(I_{cX},I_{cZ})	the moment of inertia of the car body around X- and Z- axis respectively
R_c	the radius of curvature with regard to the centroid of the car body
ϕ_c	the angle of superelevation corresponding to the centroid of the car body
$\ddot{\varphi}_c$	the second-order derivative of ϕ_c
$\dot{\varsigma}_c$	the first-order derivative of the track curvature
H_{tw}	the vertical distance between the centroid of the bogie frame and the center of the wheelset
H_{bt}	the vertical distance between the centroid of the bogie frame and the bottom plane of the secondary suspension
H_{cb}	the vertical distance between the centroid of the bogie frame and the upper plane of the secondary suspension
(k_{sz},k_{sy},k_{sx})	the secondary suspension stiffness in vertical, lateral and longitudinal directions
(k_{pz},k_{py},k_{px})	the primary suspension stiffness in vertical, lateral and longitudinal directions
(d_s,d_p)	the semi-horizontal distance of the secondary and primary suspension respectively
l_c	the semi-longitudinal distance between bogies
l_t	the semi-longitudinal distance between wheelsets in a bogie
(l_h,l_v)	the lateral and vertical distance between the wheel-rail contact point and the centroid of the rail

search scopes since the issues on lateral stability and safety of system components have gradually become notable concerns in railway engineering, especially in conditions of high speed operations. Accounting for this, more and more researchers start to focus on building the three-dimensional train-track/bridge interaction models, see for example, Zhai et al. [9] made a pioneering work in comprehensively considering the coupled dynamics between a vehicle and the tracks, in which the