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An interval inverse method based on high dimensional model representation and affine arithmetic

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ABSTRACT

This paper proposes an interval inverse method through a high dimensional model representation and affine arithmetic, which can effectively solve inverse problems with interval uncertainty. Firstly, when only the bounds of responses can be obtained from a limited number of experimental measurements, an interval model can be employed to describe the uncertainty of the measured responses and identified parameters. Secondly, in order to reasonably estimate the degree of the closeness between the measured and calculated responses, an error interval and corresponding optimization model are constructed for the interval inverse problem. Thirdly, a high dimensional model representation is utilized to approximate the original system model, and an affine arithmetic is adopted to efficiently calculate the response bounds. Finally, the optimization model for interval inverse problem is solved using genetic algorithm to identify the upper and lower bounds of the system parameters. Three examples are studied to demonstrate the correctness and effectiveness of the proposed method.

1. Introduction

In contrast with forward problems, inverse problems can be defined as the problems to identify the unknown parameters through comprehensive utilization of the measured responses and simulation models. In engineering application, many problems can be classified as inverse problems, such as the parameter estimation of material characteristic [1], load identification [2], and identification of structural flaws and cracks [3]. Meanwhile, uncertainties are generally existing in the practical engineering, for example the measured responses are accompanied with considerable uncertainty which may be caused by inaccurate measurements or inherent uncertainty of structural parameters. Thus, it is quite important to develop accurate inverse method for evaluating the identified results, in which various uncertain factors are taken into account.

With the rapid development of computational methods and simulation technologies, deterministic inverse problem has gained extensive attention and obtained a large number of research achievements. In general, the deterministic inverse methods can be divided into two categories, namely, the gradient-based method [4–6] and intelligent computational method [7–9]. For the gradient-based method, a descent direction of the objective function is searched step by step using an

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appropriate step size, in which the values of parameters can be modified in the inverse model to meet the condition of convergence in the inverse objective function. However, this kind of method need to calculate derivatives and cannot guarantee global convergence. In order to overcome these shortages, another kind of intelligent computational method such as genetic algorithm (GA) was developed to deal with such problems, which can not only provide simplicity to calculate function values in the forward model but also have strong global search capability.

With deeper study on the inverse problem, uncertainty began to draw the attention of researches. At present, most researches on uncertain inverse problem are carried out on the basic of probabilistic metric, and the Bayesian method and maximum likelihood estimation method are the two most widely used approaches to deal with the uncertain inverse problem. Tarantola [10] made a systematic analysis on uncertain inverse problem based on Bayesian theory. Fonseca et al. [11] realized uncertainty identification utilizing the maximum likelihood method combined Markov Chain Monte Carlo (MCMC) with perturbation method. Nichols et al. [12] investigated a Bayesian approach to approximately obtain the parameter distributions of nonlinear structural systems and applied the MCMC method to take samples of the posterior parameter distributions. In order to improve the efficiency of Bayesian approach, Zhang et al. [13] presented a fast sampling algorithm for the posterior probability distribution of the unknown structural parameters. Liu et al. [14] applied a point estimation and maximum entropy principle to identify the structural parameters with uncertainties. Nowak [15] adopted the Bayesian approach to identify the parameters in the thermos-acoustic problem. However, in the practical problems, the experimental and measured data are usually difficult to acquire, which makes it is hard to obtain the exact probability density functions. In addition, there are investigations indicating that even a small deviation of the probability distribution could result in a large error during uncertainty analysis [16]. In such a case, the probabilistic method is not applicable to solve this kind of inverse problem.

Compared with the probability-based method, interval analysis method [17,18] is more convenient and economical, which can be applied to describe the structural uncertainty when experimental data are limited. In recent years, several interval analysis methods were presented to solve such inverse problem with interval parameters. In the interval analysis method based on interval arithmetic [19], overestimation is serious due to the intrinsic wrapping effect in the conventional interval computations. In order to reduce the overestimation, Qiu and Wang [20] proposed an interval analysis method based on the Taylor expansion to obtain dynamic responses of structures. Wu et al. [21] presented an interval analysis method based upon the improved first-order Taylor expansion to obtain static or dynamic responses of uncertain structures. Qiu and Wang [22] used a parameter perturbation method to evaluate the range of dynamic responses of uncertain structures. In order to estimate the ranges of nonlinear dynamic responses, Qiu et al. [23] also proposed a non-probabilistic interval analysis method on the basic of the second-order Taylor expansion. Hu and Qiu [24] combined convex model with interval analysis to identify the bounds of dynamic responses for uncertain structures. Wu et al. [25] proposed an interval analysis method for dynamic responses of nonlinear systems using the Chebyshev polynomial, in which the truncated Chebyshev expansion can control the overestimation during interval computations. Liao et al. [26] applied the interval analysis method to solve power flow problem.

At present, the related systematic research on the interval inverse problem is rather less. The main idea of this study is to transform the interval inverse problem into several deterministic inverse problems or optimization problems for evaluating the interval of identified results. Jiang et al. [27] proposed an inverse method based on the first-order Taylor expansion to deal with the uncertain inverse problem and applied it to identify the material characterization of composite laminates. Liu et al. [28] presented an inverse method that combines the interval analysis with regularization to identify the bounds of dynamic loads stably. Zhang et al. [29] proposed a computational inverse technique based on the interval analysis method to determine the encounter condition through a given projectile state in the projectile penetrating multilayer medium system. Feng et al. [30] presented an interval inverse analysis method based on the Chebyshev inclusion function to design the suspension of vehicle vibration model with six conflicting objective functions. Nevertheless, the above-mentioned methods are based on the first-order Taylor expansion, interval extension operation or Chebyshev inclusion function, but they are only suitable for inverse problem with low uncertainty level.

In this study, a computational inverse method is proposed on the basic of the high dimensional model representation (HDMR) [31,32] and affine arithmetic (AA) [33,34] in order to more effectively solve the inverse problem with large uncertainty level. The HDMR is utilized for system model approximation which can bring convenience to calculate the response bounds using the AA. The proposed interval inverse objective function is modeled based on the defined error interval and then GA [35] is employed as a solver to obtain the upper and lower bounds of the identified parameters. The plan of the remaining of this paper is given as follows. In Section 2, the interval inverse problem and corresponding inverse optimization model based on the defined error interval are described. In Section 3, an interval inverse method based on the HDMR and AA is presented. In Section 4, three examples are provided to show the high efficiency and accuracy of the proposed inverse method. Finally, some conclusions are drawn in Section 5.

2. Modeling for interval inverse problem

An interval X^I can be defined as follows,

$$X^I = \left[X^L, X^U\right] = \left\{X \middle| X^L \le X \le X^U\right\} \tag{1}$$