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A new measurement for structural uncertainty propagation based on pseudo-probability distribution

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ABSTRACT

Uncertainty propagation (UP) offers a powerful tool for describing uncertainties from input parameters to output responses in a system. The existing methods of non-probabilistic UP can only evaluate the upper and lower bounds of structural responses. In this study, a new non-probabilistic UP method is proposed, attempting to provide more detailed quantification of uncertain responses between the lower and upper bounds. A concept of pseudo-probability distribution is proposed under the non-probabilistic UP frame to quantify the possibilities of system responses. The uncertainties of structural parameters are modeled as a multi-dimensional ellipsoid convex set in the proposed UP method. The ellipsoid domain is divided into two parts using the first-order approximation of the system-state function. Then the volume ratio of the divided domain and the whole ellipsoid domain can be used to calculate the pseudo-probability of system responses. The sequential improved Hasofer-Lind-Rackwitz-Fiessler (*i*HL-RF) algorithm is adopted to effectively obtain the most probable expansive point of system-state function. The proposed UP method can not only provide accurate response bounds, but also objectively quantify the relatively accurate possibilities of each response value. In the numerical examples, the proposed UP method is compared with the Monte Carlo simulation method and traditional non-probabilistic uncertainty propagation method, and the calculated results demonstrate the validity and effectiveness of the proposed UP method.

1. Introduction

Uncertainty propagation (UP) and qualification plays an important role in predicting the behavior of uncertain problems [1–3]. For engineering problems, the uncertainties commonly arise from material properties, external loads, manufacturing errors, boundary conditions, etc. In most cases, the ranges of these uncertain parameters are relatively small, but the coupling among them may lead to the non-ignorable deviations in system responses [4,5]. In order to effectively evaluate the behaviors and influences of the uncertainties of input parameters on system responses, at present, some kinds of UP approaches including probabilistic approach [6–9], convex set approach [10,11], fuzzy approach [37] and hybrid approach [12] have been developed.

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The probability model, as a traditional mathematical model in uncertainty analysis, has been widely adopted in varieties of engineering problems [13,14], and a series of methodologies based on probabilistic frame have been well established [15–17]. However, the probabilistic method is developed based upon a precise probability density function (PDF) of uncertain parameter, which requires a large amount of sample data. Unfortunately, the sufficient samples are frequently unavailable or expensive in many practical engineering problems. To deal with the uncertain problems with limited sample information, Ben-Haim [18], Ben-Haim and Elishakoff [19] proposed an UP methodology based on the non-probabilistic convex model, in which the variation bounds of uncertain parameters were only required and the fluctuations of uncertain parameters were assumed to fall into a convex set. Thus, this non-probabilistic method could be made possibly to deal with the complex engineering problems with limited samples. Since then, convex model theory entered a period of rapid development. Qiu and Elishakoff [20] proposed an UP method for static and dynamic engineering problems. McWilliam [21] proposed two approaches for determining the least favorable displacement response of structure with uncertain parameters. Ganzerli and Pantelides [22] presented a method for implementing a multi-dimensional interval convex model for the optimal deign of structure with the uncertain-but-bounded loads. Jiang et al. [23] developed a correlation analysis technique for uncertain parameters and a novel mathematical method was proposed to establish the multi-dimensional ellipsoid convex model for uncertainty analysis. Liu et al. [4] applied the interval convex model to identify the dynamic load of uncertain structures in the inverse UP problem. Moreover, when the probabilistic and non-probabilistic parameters exist in practical engineering problems, a hybrid uncertain model can be used to deal with uncertainties. In the recent years, some efforts on hybrid uncertainty model have been reported [24–27]. For example, the random variables with interval parameters were used to deal with the uncertain parameters lacking of sufficient experimental data [24] and the hybrid uncertainty model was extended to solve a system reliability problem [25]. In the above mentioned studies, the uncertainty qualification by the convex model plays an important role, just like the PDF of uncertain parameters in the probabilistic frame.

However, almost all the UP methods based upon the non-probabilistic model only focused on calculating the lower and upper bounds of responses, and more detailed quantification of uncertain responses between the lower and upper bounds such as the distribution status of responses in the interior of interval cannot be provided, which may bring inconvenience for designers or lead to too conservative design scheme. In fact, in order to address this issue some other uncertainty measures, e.g., fuzzy uncertainty and grey uncertainty, have been proposed [38]. For example, a man-made subjective membership function is given in the fuzzy uncertainty measure to describe the distribution status of response in the interior of interval. Yet, although the fuzzy and grey uncertainty measures tempt to quantify the response distribution in the interior of interval, the quantifications still are the subjective measure. In order to more objectively and reasonably quantify the possibility of response in the interior of interval, a concept of pseudo-probability distribution based on ellipsoidal convex model is firstly proposed in this study.

In order to realize the objective quantification of the distribution status of uncertain response in the interior of interval, the uncertainties of structural parameters are modeled by non-probabilistic ellipsoidal convex model [28] and the uncertain responses are calculated using the sequential *i*HL-RF algorithm [29]. Meanwhile, a new pseudo-probability measurement as pseudo-cumulative distribution function (p-CDF) is proposed by using the volume ratio [29,30] between the divided domain and whole domain. The proposed UP method in this paper cannot only accurately obtain the lower and upper bounds of response, but also comprehensively evaluate the objective distribution status of response in the interior of interval. The remainder of this paper is organized as follows. Section 2 describes the problem of UP using the ellipsoidal convex model. Section 3 provides a traditional non-probability UP method which only can obtain the bounds of uncertain response. By contrast, a new measure named p-CDF is proposed to obtain a more comprehensive and objective pseudo-probability distribution in the response interval, and the sequential and inverse *i*HL-RF algorithm is adopted in Section 4. Section 5 presents three numerical examples to demonstrate the effectiveness of the proposed UP method and some conclusions are finally summarized in Section 6.

2. Problem description

For the actual engineering problem, it might be extremely difficult or expensive to obtain sufficient samples of uncertain parameters and corresponding precise PDF. In order to overcome this shortcoming, several non-probabilistic convex models were gradually developed and applied to the problem of UP. Ellipsoid is one of the most popular convex models, which will be adopted to describe the uncertainties of input parameters in this study.

Assume that $\boldsymbol{X}=\{X_1,X_2,\ldots, X_n\}$ is an n-dimensional vector including all the uncertain parameters of a structure. For each parameter X_i, the possible values can constitute a bounded interval which is denoted as $\boldsymbol{X}^I=[\boldsymbol{X}^L,\boldsymbol{X}^R]$, where the superscripts *I, L* and *R* represent the interval, lower bound and upper bound, respectively. Then the whole uncertain domain E_X of $\boldsymbol{X}$ can be modeled using a multi-dimensional ellipsoid [23],

$$E_X=\left\{X\left|\left(X-X^C\right)^T\Omega_X^{-1}\left(X-X^C\right)\le 1\right.\right\} \tag{1}$$