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Modeling of three dimensional thermocapillary flows with evaporation at the interface based on the solutions of a special type of the convection equations

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ABSTRACT

Theoretical and numerical study of the convection processes, which are accompanied by evaporation/condensation, in the framework of new non-standard problem is largely motivated by new physical experiments. One of the principal questions is to understand the character and to evaluate the degree of influence of particular factors or their combined action on the structure of the joint flows of liquid and gas-vapor mixture. The flow topology is determined by four main mechanisms: natural and thermocapillary convection, tangential stresses and mass transfer due to evaporation at the interface. The mathematical modeling of the fluid flows in an infinite channel with a rectangular cross section is carried out on the basis of the solution of a special type of the convection equations. The effects of thermodiffusion and diffusive thermal conductivity in the gas phase and evaporation at the thermocapillary interface are taken into consideration. Numerical investigations are performed for the liquid – gas (ethanol – nitrogen) system under normal and low gravity. The fluid flows are characterized as translational and progressively rotational motions and can be realized in various forms.

1. Introduction

Flows of evaporating liquids with an interface being under the action of tangential stresses have been the subject of numerous experimental [1–4] and theoretical [5–9] investigations during the last decade. Great interest in such two-phase systems is explained by the necessity of finding innovative technical solutions for some problems of thermophysics (thermostabilization and cooling set-ups, thermal management of advanced semiconductor devices), engineering (coating applications and drying processes, microfluidic biochips or membrane functioning, distillation) and optimization of fluidic technologies, using evaporating liquids and vapor-gas mixtures as the working media.

Physical experiments are an important motivation in deriving a strong unified mathematical theory for describing the evaporative convection, obtaining the governing parameters to identify the cases of weak or intense evaporation and for defining the conditions of evaporation or condensation, and finally, for improving our understanding of the fundamental aspects of convection in the domains with the mass transfer at the interfaces [8,10–14]. New physical experiments performed

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in certain scientific projects of the European Space Agency allow one to investigate the structure, intensity and topology of flows arising under the terrestrial conditions and under microgravity. The fluid flow patterns are determined by various thermal, mechanical and structural effects. Among all mechanisms defining the flow structure the following factors are especially specified by researchers: natural convection and thermophysical properties of the media, tangential stresses induced by a co-current gas flow [2–4,7,9,15], thermocapillary forces [5,10,16–18] and evaporation/condensation effects [3,4,6,10,15,19,20].

From the mathematical point of view a combination of different effects significantly complicates the scientific problem. At the moment, there is no general universal mathematical theory exhaustively describing the motion of two-phase systems and taking into account phase composition, phase concentration, the interaction between particles, turbulent pulsations of the medium and potential phase transitions. The physics and chemistry of these processes involve heat, mass and momentum transfer. Studies of these are based on the fundamental laws of classical mechanics, continuum mechanics, physical chemistry and thermodynamics. Most of the above-mentioned theoretical and numerical investigations were performed in the framework of the Navier – Stokes equations or their Oberbeck – Boussinesq approximation, which imply the natural properties of symmetry of the space - time and of a fluid moving in the space. The equations possess rich group properties [21]. These properties have multifarious manifestations and the most important of them is the possibility of obtaining the exact solutions of the equations. The role of exact solutions is important at all stages of theory development. Exact solutions are indispensable for testing numerical methods, analysis of singularities in solutions, development and verification of different approximate models. Lastly, exact solutions have an applied significance; one has only to consider the Poiseuille formula or Couette viscometer. Studying the solutions of a special form of original equations, we refer to [22], where sufficiently complete consideration of the concept of "exact solution" has been carried out. In the classical sense an exact solution is a solution that is written in the form of perfect formulae, quadratures, series or special functions. According to [22] a set of exact solutions of the hydrodynamics equations can be extended by invariant and partially invariant solutions of rank of 1 or 2. Most of the exact solutions of the Navier – Stokes or Oberbeck – Boussinesq equations possess an invariant property with respect to some group of transformations admitted by these basic systems. Thus, the solutions allow one to effectively study the fundamental (or secondary) features of the physical processes described with the help of the Navier – Stokes or Oberbeck – Boussinesq equations. They conserve the symmetry properties provided by the derivation of the equations.

One of the first solutions of the governing equations in the problem on two-layer flows with the mass transfer at the "liquid – liquid" interface was obtained in [23]. In [24,25] the exact solutions describing forced flows of liquids, undergoing a phase change, were constructed in the case of a given specific gas flow rate [25] and under the closed flow conditions in each phase [24]. These solutions can be considered to be an analogue of the well-known Ostroumov – Birikh solution [26,27] in the case of the thermoconcentration-induced convection equations. The group nature of the Birikh solutions was analyzed in [28]. In [29] a three dimensional exact solution was constructed to describe two-layer fluid flows in an infinite channel in the non-axisymmetric case. The analysis and classification of the analogue of the Ostroumov – Birikh solution, depending on the boundary conditions for vapor concentration, flow topology, structure of the temperature field and the inclusion/exception of the Soret effect, were presented in [9]. Furthermore, the stability of a two-layer flow with evaporation, modeled using one of the possible classes of the constructed solutions, was investigated in the last work. A comparison of the experimental and theoretical results was made [8]. Theoretical, numerical and experimental studies of the problem were performed for the cases of ethanol and HFE-7100 (HydroFluoroEther-7100, high-performance segregated dielectric) as working fluids and nitrogen or air as inert gas (see, for instance, [4,8,9]).

In contrast to the exact solutions aimed at explaining the fundamental features of the phenomena, numerical investigations, including direct numerical simulation (DNS), are utilized in the problems where an analytical solution is unavailable. As a rule, such a necessity arises in real practical problems. 3D DNS of two-phase dynamics of thermocapillary flows in a rectangular-section cuvette with an applied thermal load was realized on the basis the Navier – Stokes and energy equations [30,31]. The influence of the Marangoni effect and liquid – vapor phase transition on the spatio-temporal flow patterns, nature of the interface deformation and stability characteristics was investigated.

The present consideration is devoted to the mathematical modeling of the evaporative fluid flows based on the 3D generalization of the Ostroumov – Birikh solution of the Boussinesq approximation of the Navier – Stokes equations. The influence of the thermal load, gravity and evaporation on the structure of the temperature and velocity fields is studied. Matters related to the stability of solution under study are the subject of a separate investigation and are not considered in the present article.

2. Problem statement

We restrict ourselves to the consideration of the stationary problem under the assumption that the interface Γ remains flat.

2.1. Basic assumptions, governing equations and boundary conditions

Consider two immiscible media (a liquid and a gas) filling the infinite horizontal domains (see Fig. 1)

$$\Omega_1 = \{(x, y, z) : -x_0 < x < 0,\ 0 < y < h,\ -\infty < z < \infty\},$$

$$\Omega_2 = (x, y, z) : 0 < x < x^0,\ 0 < y < h,\ -\infty < z < \infty\}.$$